Computer Graphics

Clipping & Hidden Surface Removal

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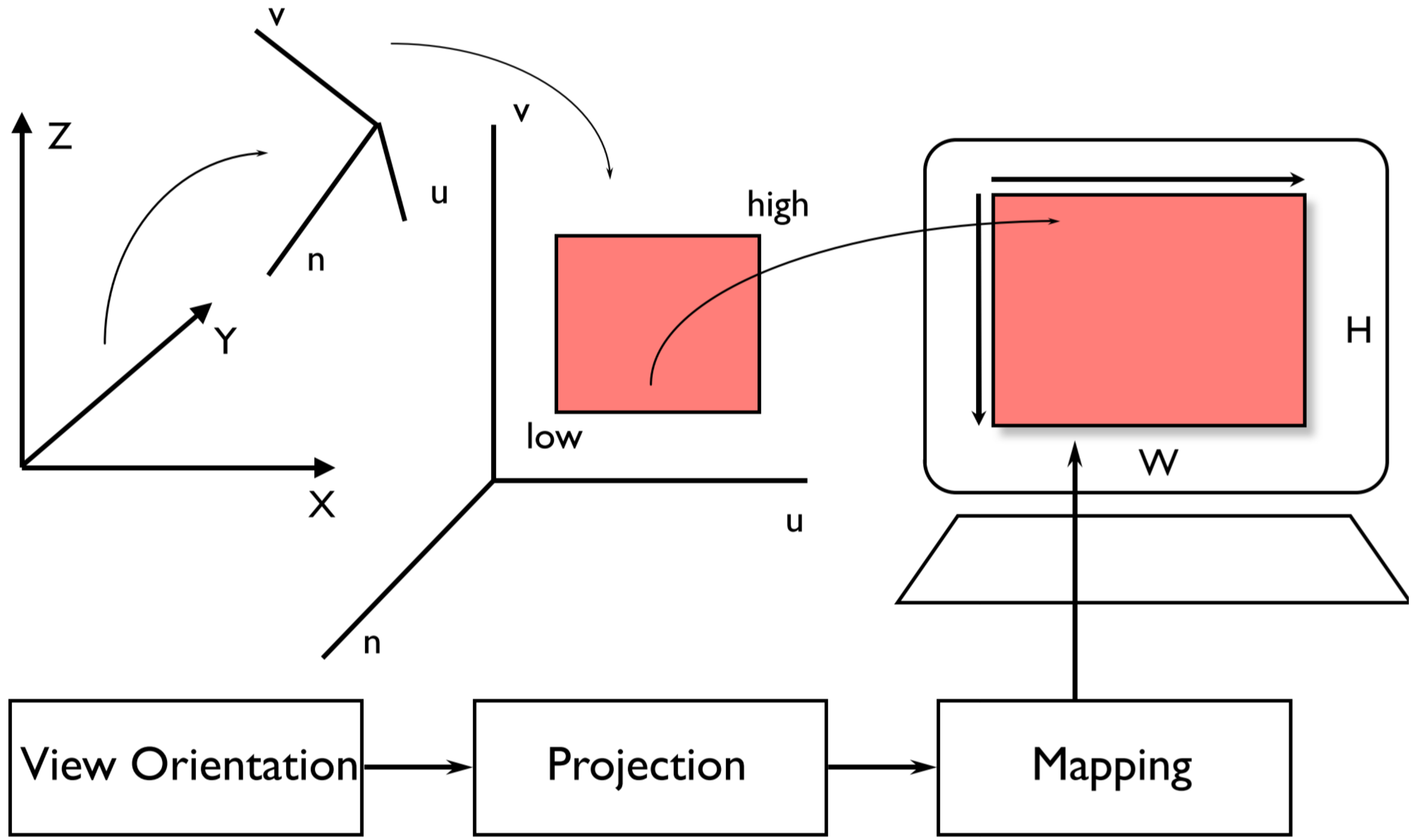
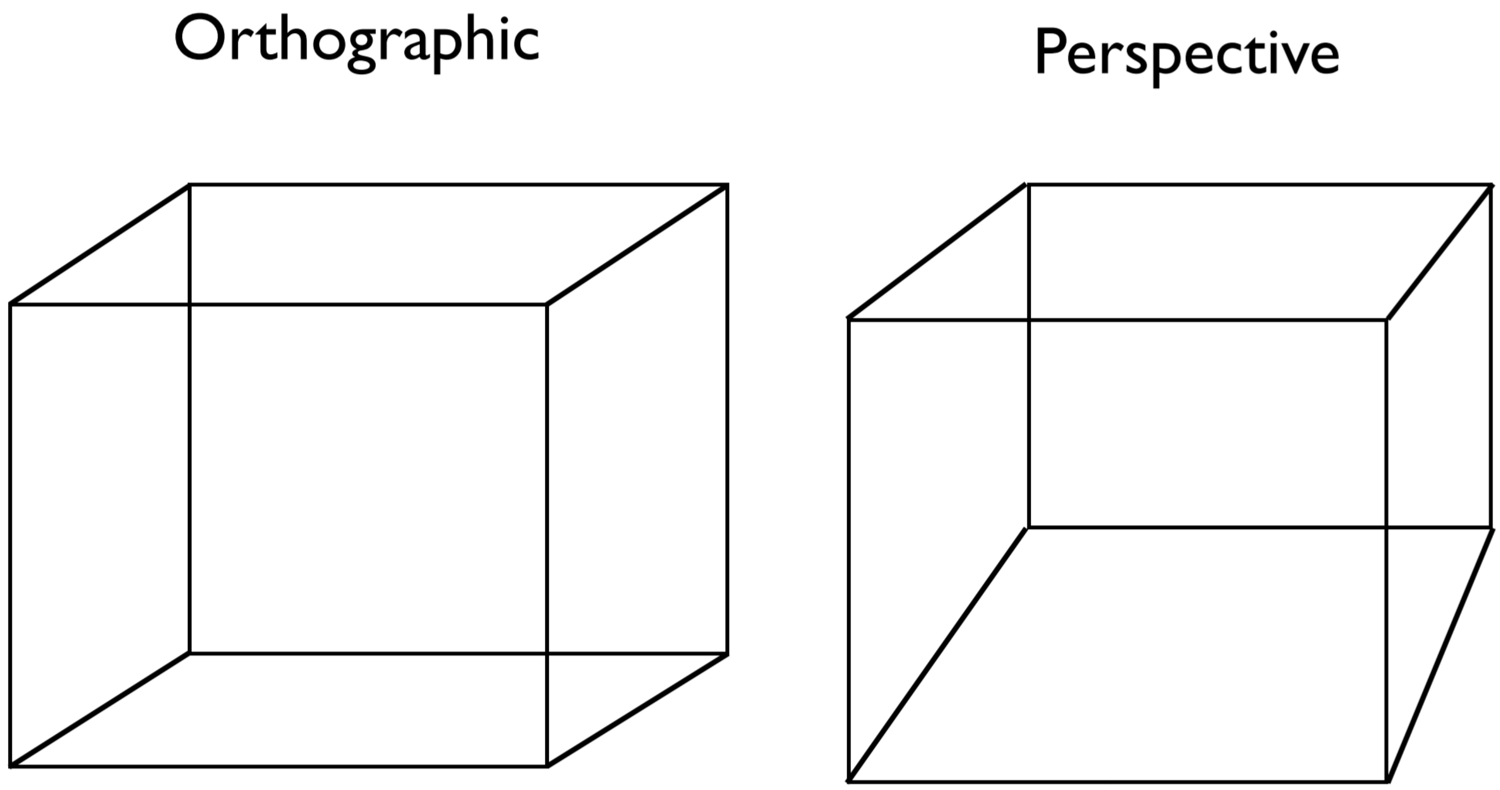
School of Data and Computer Science

Viewing Pipeline Review



Projection

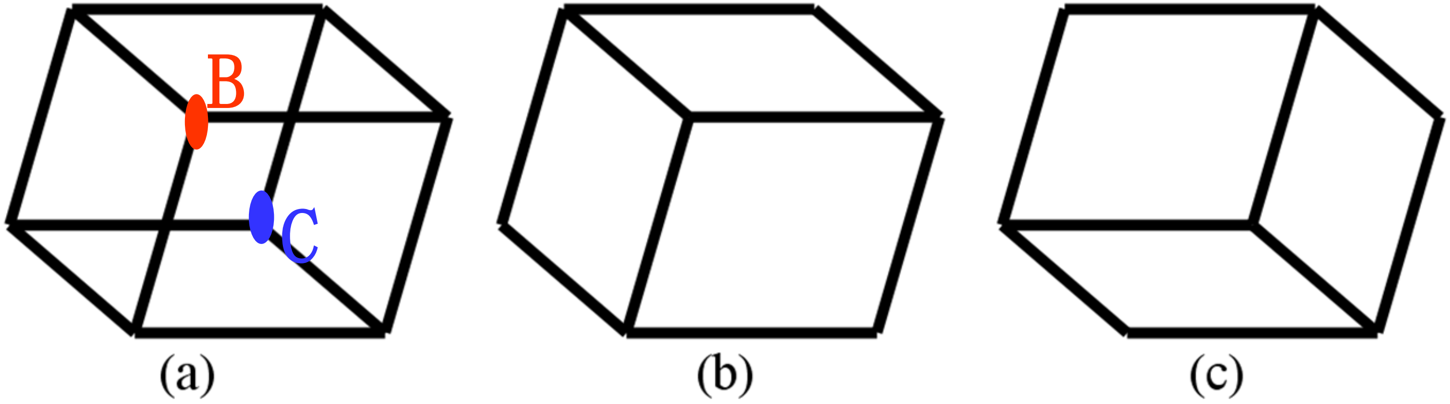


Computer Graphics @ ZJU

Hongxin Zhang, 2015

Why eliminating invisible objects?

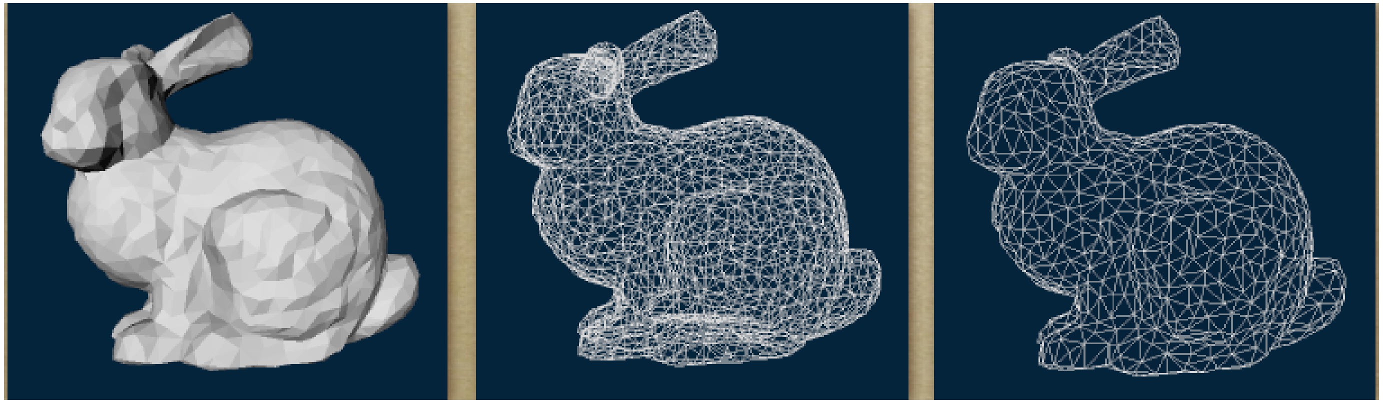
- Hidden surface removal (HSR) may reduce ambiguity



(a) Cube wireframe; (b) B is the nearest; (c) C the neartest

Why eliminating invisible objects?

- **Visible** and **invisible** portions of objects
- Enhance reality (增加图形的真实感)
  - Projection： 3D space→2D space
  - 2D space： sorting according to depth may add 3D cueing

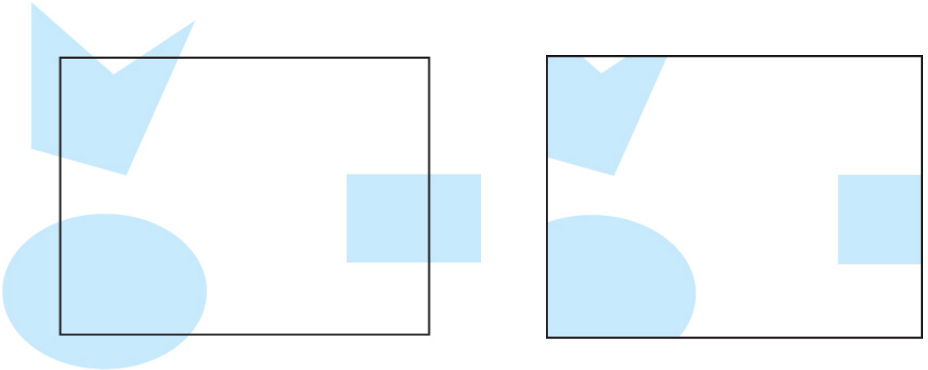


Visible Surface Determination

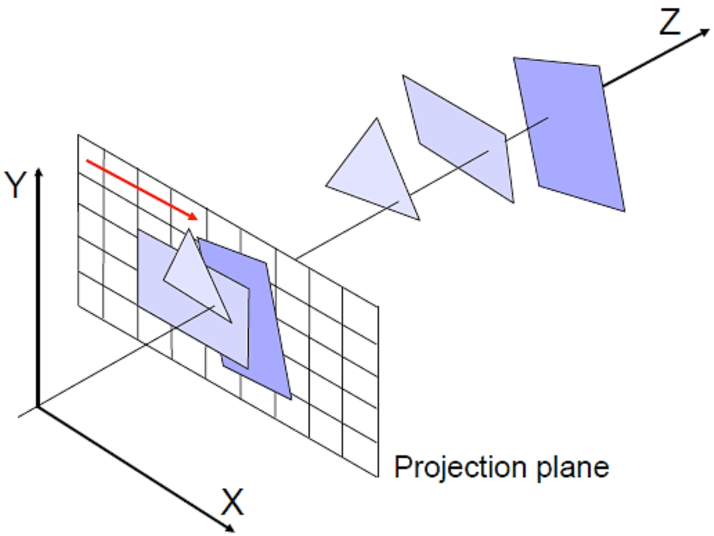
- Goal
  - Given: a set of 3D objects and Viewing specification,
  - Determine: those parts of the objects that are visible when viewed along the direction of projection
- Elimination of hidden parts (hidden lines and surfaces)
- Visible parts will be drawn/shown with proper colors and shade

Outline

- Clipping



- Hidden Surface Removal

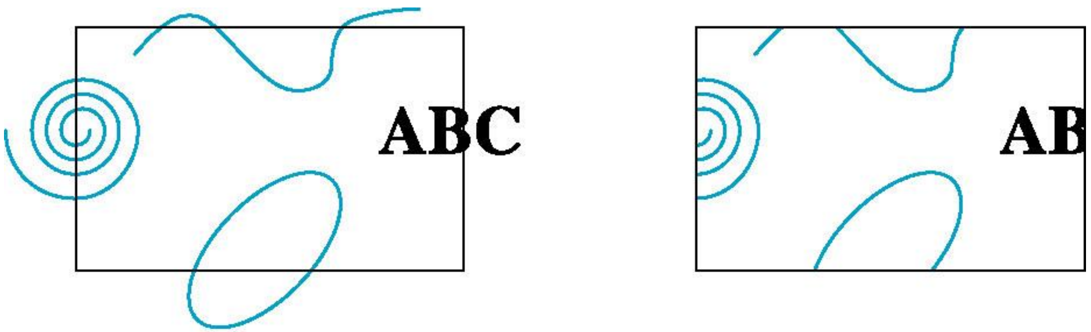


Clipping

- Clipping of primitives is done usually **before** scan converting the primitives
- Reasons being
  - Scan conversion needs to deal only with the clipped version of the primitive, which might be much smaller than its unclipped version

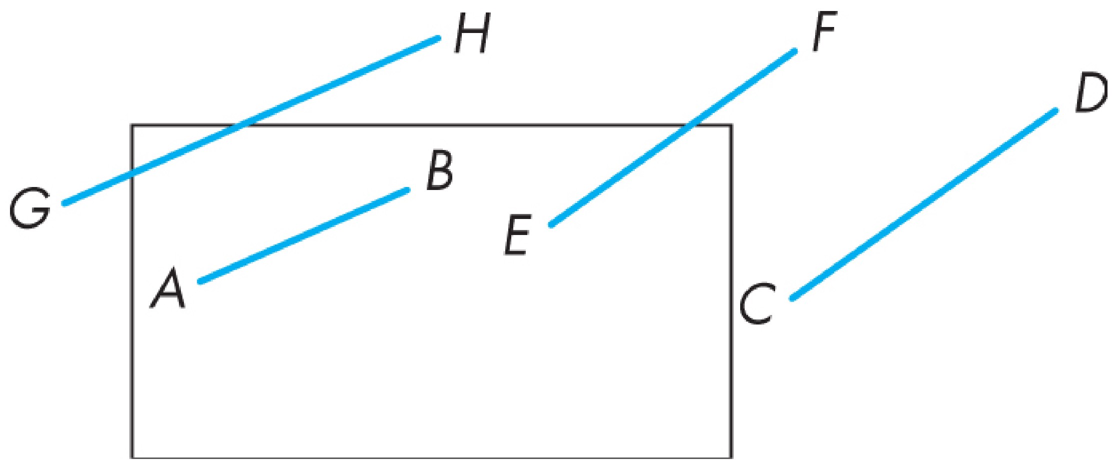
How would we clip?

- 2D clipping
- Clipping is easy for Line and Polygons
- Clipping is hard for curve and Text
  - They can be converted to lines and polygons first



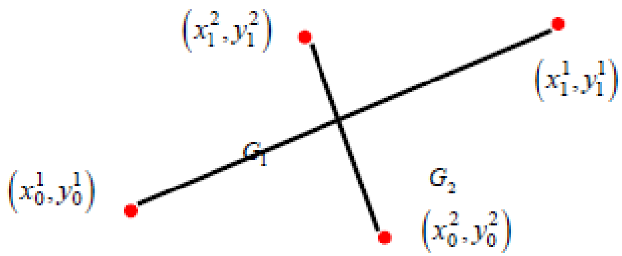
2D Clipping Methods

- Brute force approach:
  - compute intersections with all sides of clipping window
- Inefficient: one division per intersection (需要计算除法)



Segment-Segment Intersection

-

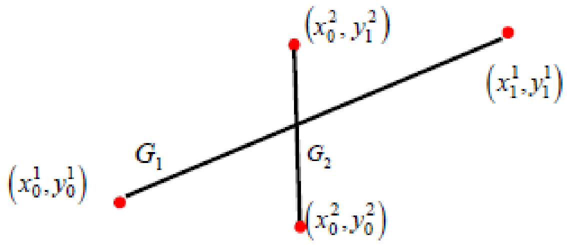


$$G_1 = \begin{cases} x^1(t) = x_0^1 + (x_1^1 - x_0^1)t \\ y^1(t) = y_0^1 + (y_1^1 - y_0^1)t \end{cases} \quad t \in [0,1] \qquad G_2 = \begin{cases} x^2(r) = x_0^2 + (x_1^2 - x_0^2)r \\ y^2(r) = y_0^2 + (y_1^2 - y_0^2)r \end{cases} \quad r \in [0,1]$$

Intersection: *x* & *y* values equal in both representations - two linear equations in two unknowns (*r*,*t*)
test if resulting r & t are inside the [0,1] range

$$x_0^1 + (x_1^1 - x_0^1)t = x_0^2 + (x_1^2 - x_0^2)r$$
$$y_0^1 + (y_1^1 - y_0^1)t = y_0^2 + (y_1^2 - y_0^2)r$$

Intersection with axis-aligned lines



$$G_1 = \begin{cases} x^1(t) = x_0^1 + (x_1^1 - x_0^1)t \\ y^1(t) = y_0^1 + (y_1^1 - y_0^1)t \end{cases} t \in [0,1], \quad G_2 = \begin{cases} x^2(r) = x_0^2 \\ y^2(r) = y_0^2 + (y_1^2 - y_0^2)r \end{cases} r \in [0,1]$$

Intersection: *x* & *y* values equal in both representations - two linear equations in two unknowns (*r*,*t*)

$$x_0^1 + (x_1^1 - x_0^1)t = x_0^2$$

$$t = \frac{x_0^2 - x_0^1}{x_1^1 - x_0^1}, \text{if } t < 0 \text{ or } t > 1 \text{ no intersection}$$

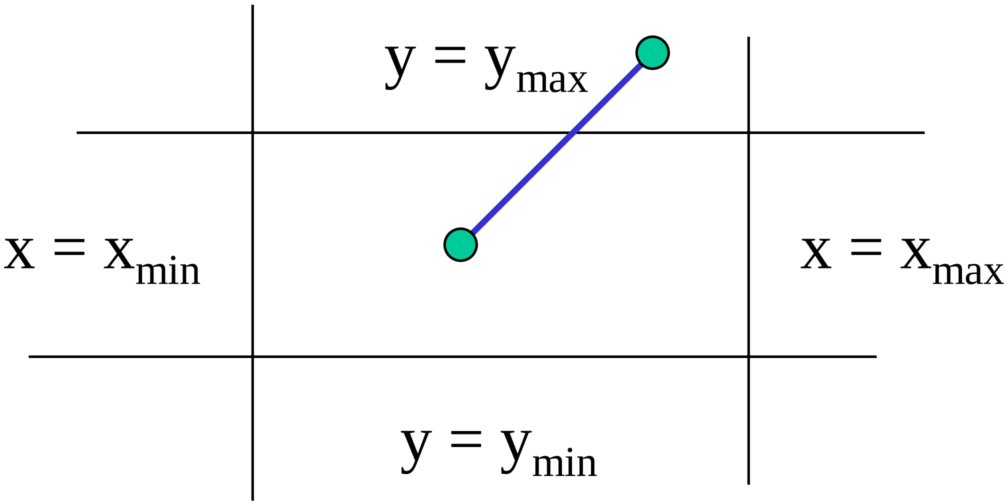
$$y_0^1 + (y_1^1 - y_0^1)t = y_0^2 + (y_1^2 - y_0^2)r, \text{ (relevant only for segments)}$$

2D Clipping Methods

- Cohen-Sutherland ： Codeing
- Mid-point clipping(中点分割裁剪)：Divided by 2，shift operation
- Parametric clipping (梁友栋-Barsky 裁剪)：High efficiency
- Nicholl-Lee-Nicholl：More precise
- ……

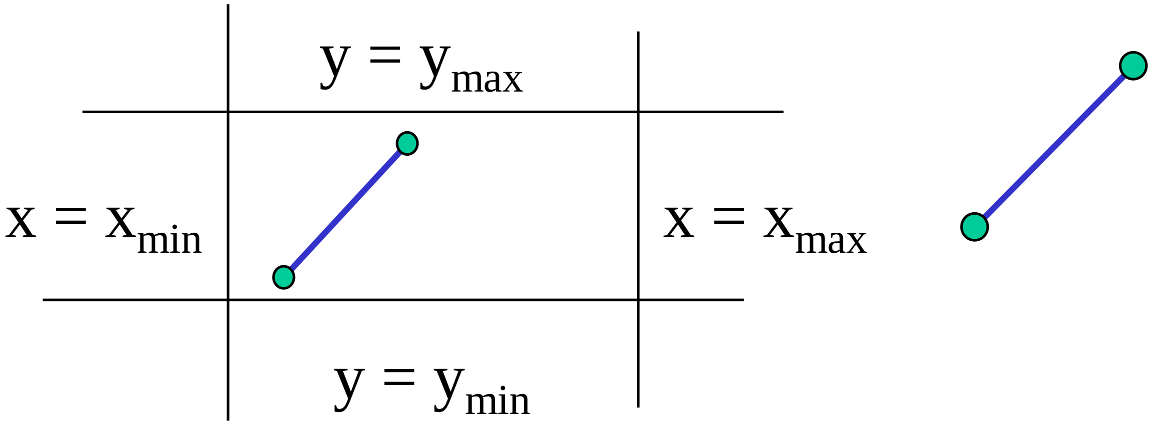
Cohen-Sutherland Algorithm

- Idea: eliminate as many cases as possible without computing intersections
- Start with four lines that determine the sides of the clipping window



The Cases

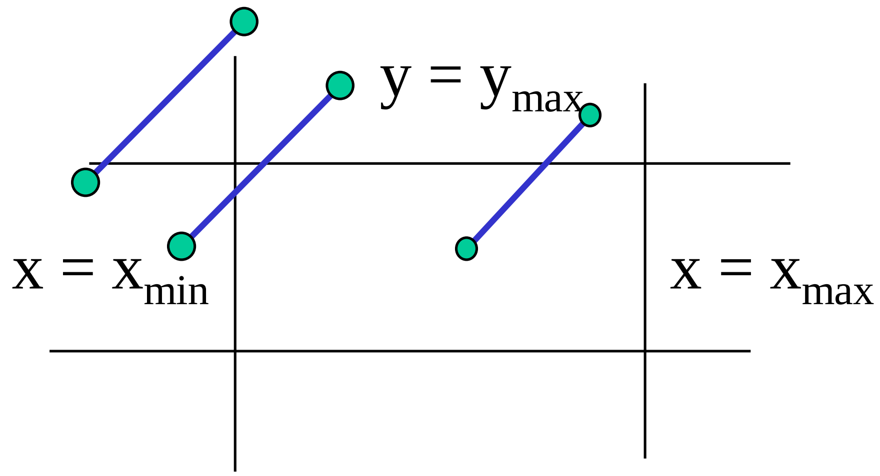
- Case 1: both endpoints of line segment inside all four lines
  - Draw (accept) the line segment as is



- Case 2: both endpoints of line segment on same side of a line
  - Discard (reject) the line segment

The Cases

- Case 3: One endpoint inside, one outside
  - Must do at least one intersection
- Case 4: Both outside
  - May have part inside
  - May the whole segment be out of windows



Defining Outcodes

- For each endpoint, define an outcode：

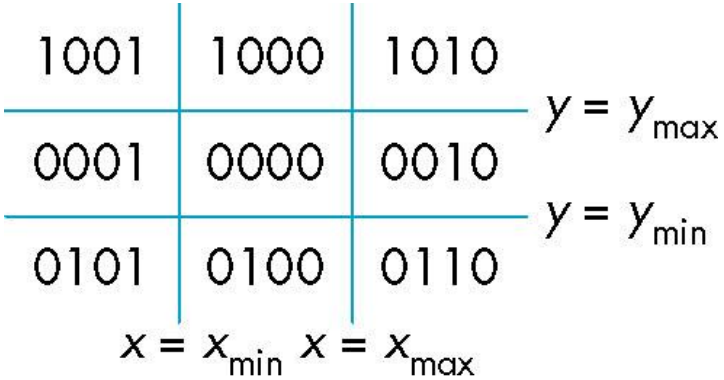
$$b_0\, b_1\, b_2\, b_3$$

$b_0 = 1$ if $y > y_{max}$, 0 otherwise

$b_1 = 1$ if $y < y_{min}$, 0 otherwise

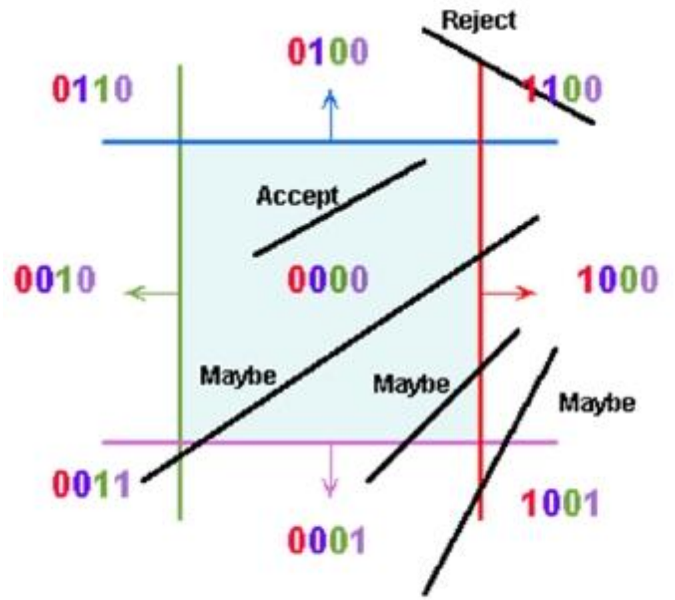
$b_2 = 1$ if $x > x_{max}$, 0 otherwise

$b_3 = 1$ if $x < x_{min}$, 0 otherwise



- Outcodes divide space into 9 regions
- Computation of outcode requires at most 4 subtractions

(outcode1 OR outcode2) == 0
line segment is inside



(outcode1 AND outcode2) != 0
line segment is totally outside

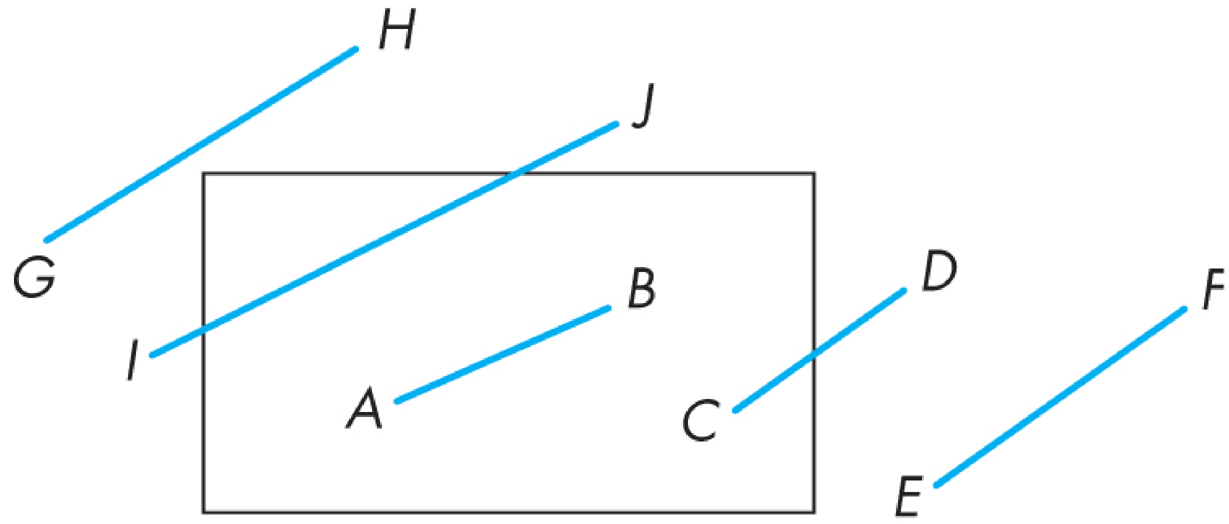
(outcode1 AND outcode2) == 0
line segment potentially crosses clip region

False positive

Some line segments that are classified as potentially crossing the clip region actually don’t

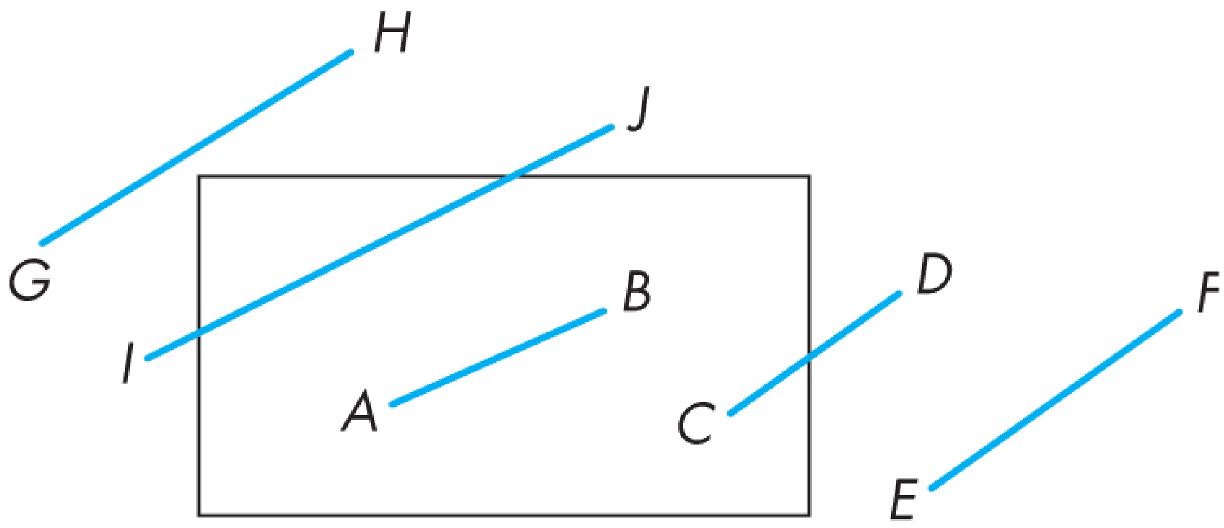
Using Outcodes

- Consider the 5 cases below
- AB: (outcode(A) OR outcode(B) == 0)
  - Accept line segment



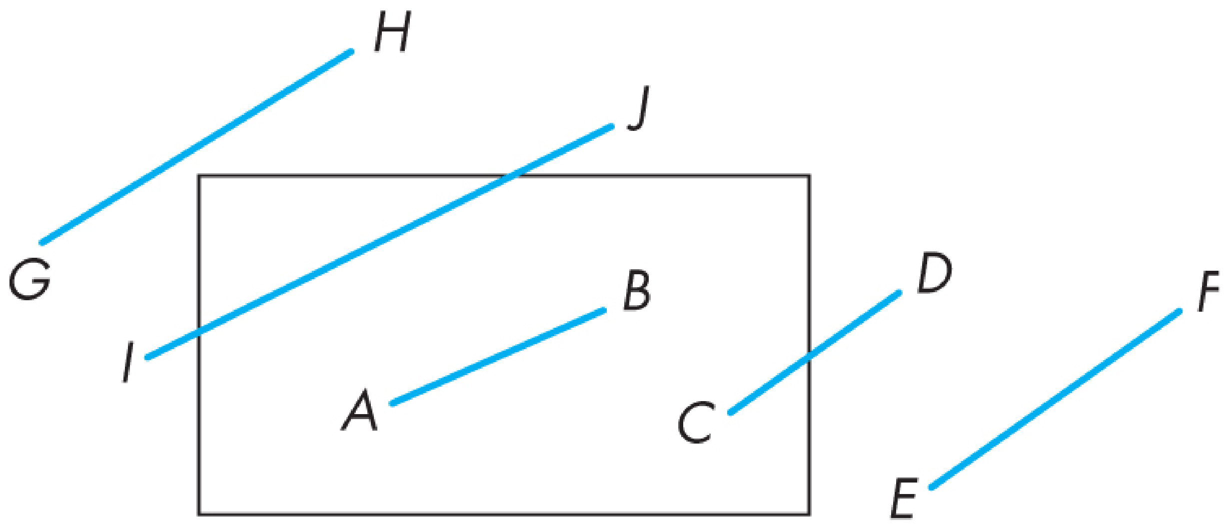
Using Outcodes

- EF: (outcode(E) AND outcode(F) ！= 0 ）
  - Both outcodes have a 1 bit in the same place
  - Line segment is outside of corresponding side of clipping window
  - reject



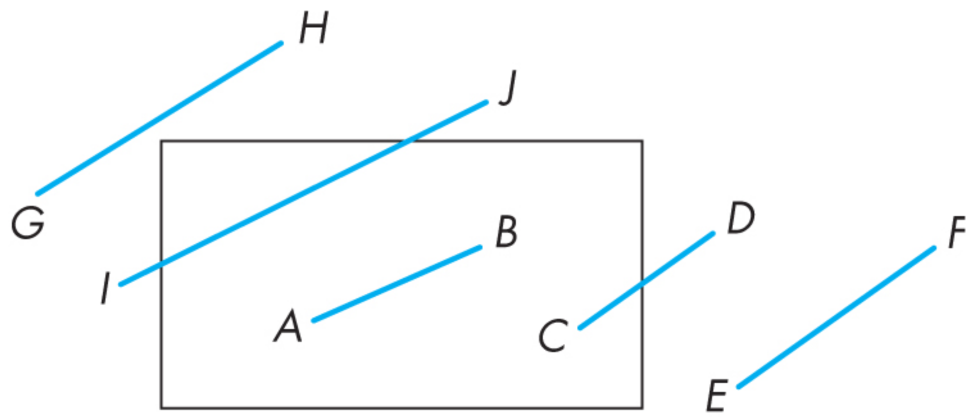
Using Outcodes

- CD: (outcode (C) AND outcode(D) == 0)
  - Compute intersection
  - Location of 1 in outcode(D) determines which edge to intersect with
  - Note if there were a segment from A to a point in a region with 2 ones in outcode, we might have to do two interesections

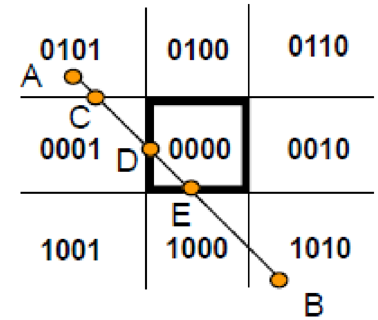


Using Outcodes

- GH and IJ: same outcodes, logical AND yields zero
- Shorten line segment by intersecting with one of sides of window
- Compute outcode of intersection (new endpoint of shortened line segment)
- Reexecute algorithm



Algorithm



Check Line P1P2:

AB → CB → DB → DE

(1) If P_1P_2 is completely inside, accept it; if P_1P_2 is completely outside, reject it; otherwise go to step 2;

(2) Find an end point P_1 (or P_2) of line P_1P_2 outside of region;

(3) Find the intersection point P'_1 to replace P_1 (or P_2)

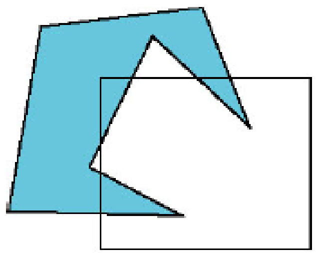
(4) If P_1P_2 is completely inside , then accept this line, else go to step 2.

Efficiency

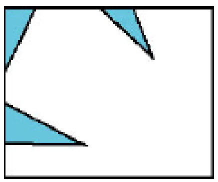
- In many applications, the clipping window is small relative to the size of the entire data base
  - Most line segments are outside one or more side of the window and can be eliminated based on their outcodes
- Inefficiency when code has to be reexecuted for line segments that must be shortened in more than one step

Polygon clipping

- It’s harder than clipping segment.
  - Clipping a segment produce a segment at most.
  - Clipping a polygon may produce several polygons.



(a)

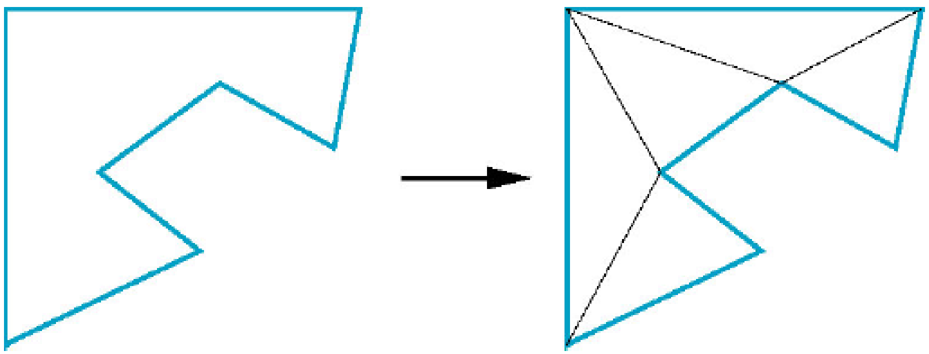


(b)

- To convex polygon, clipping a polygon only produces a polygon.

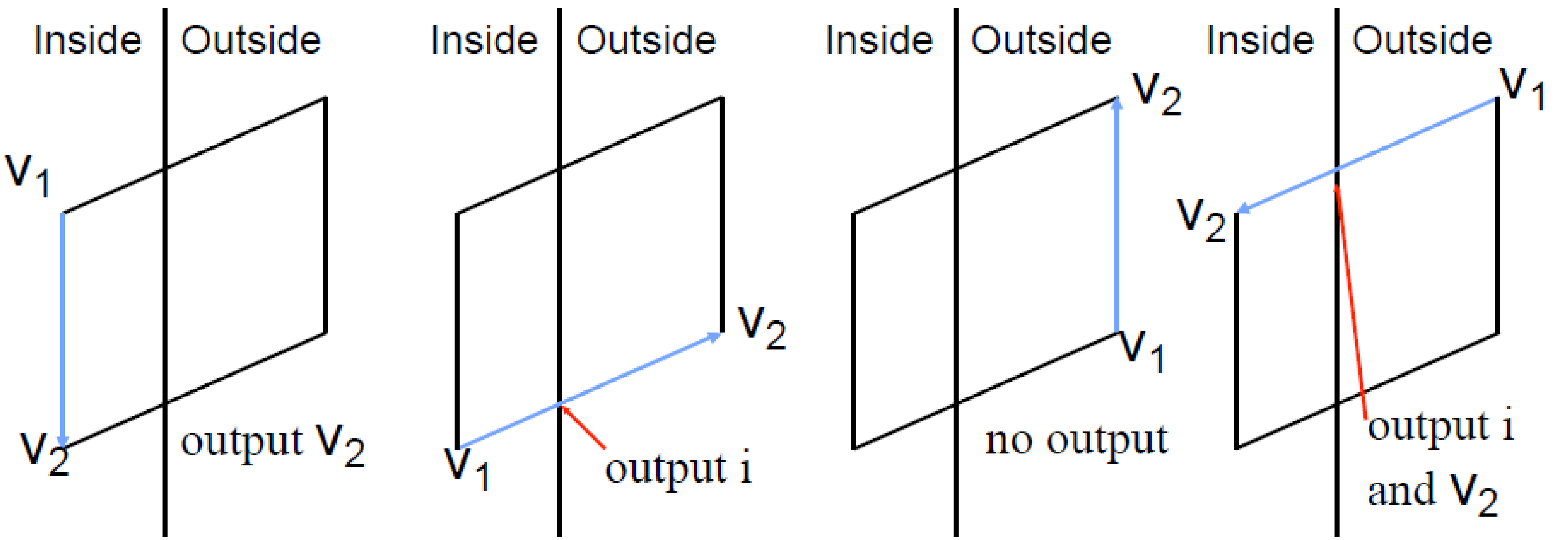
Polygon clipping

- 一种方法就是把非凸(凹)多边形用一组三角形代替，这个过程称为划分（tessellation）
- 这同样也使得填充变得简单
- 在GLU库中有划分代码，但最好的方法就是由用户自己进行



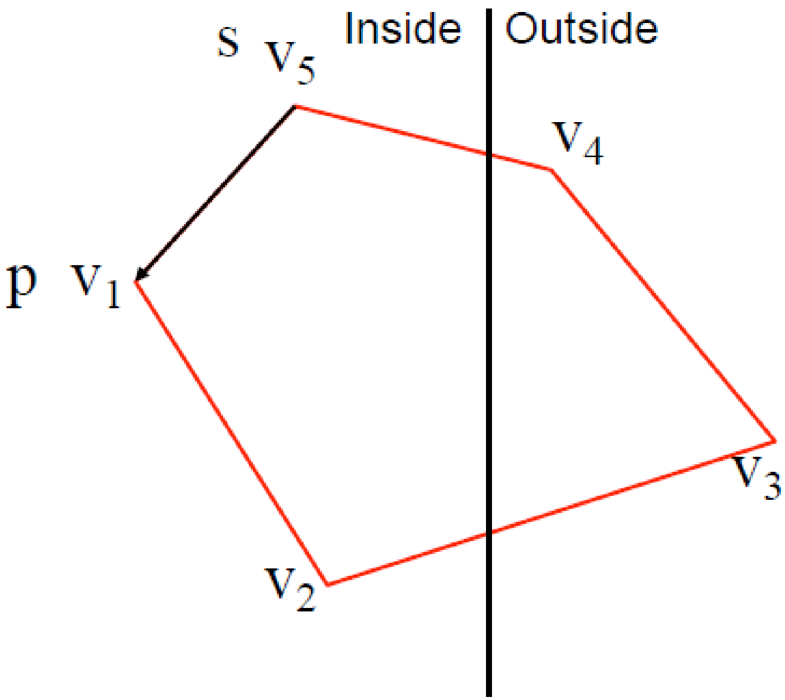
Sutherland-Hodgeman algorithm

- Present the vertices in pairs
  - (v_n, v_1), (v_1, v_2), (v_2, v_3), …, (v_{n-1}, v_n)
  - For each pair, what are the possibilities?
  - Consider v_1, v_2

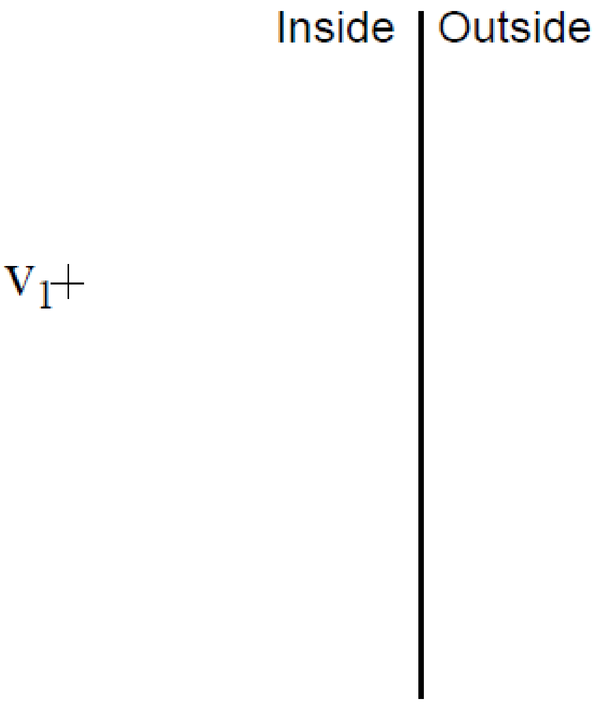


Example

v_5, v_1

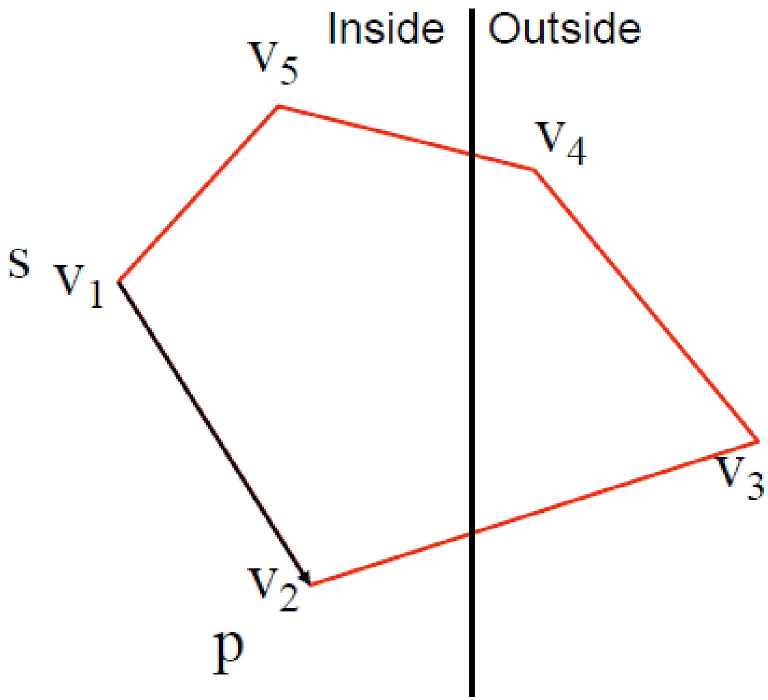


Inside, Inside
Output v_1

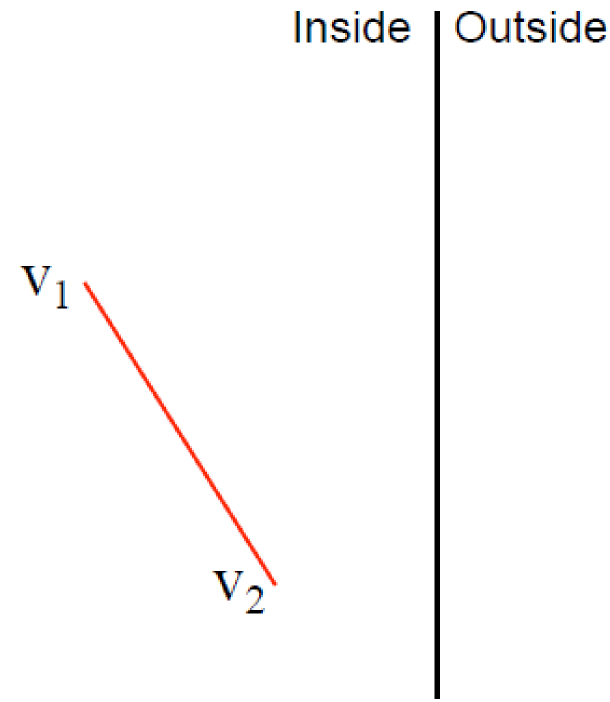


Current
Output

v_1, v_2

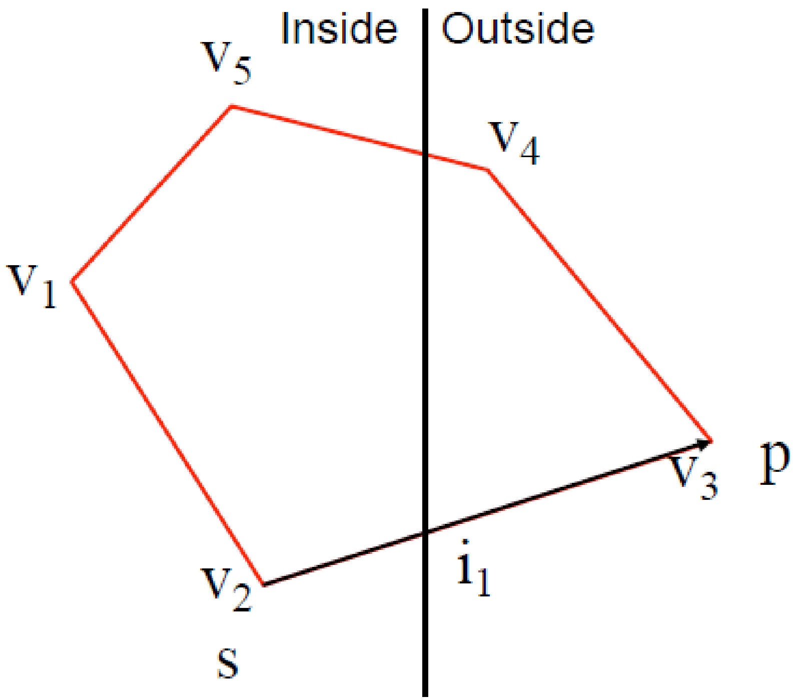


Inside, Inside
Output v_2

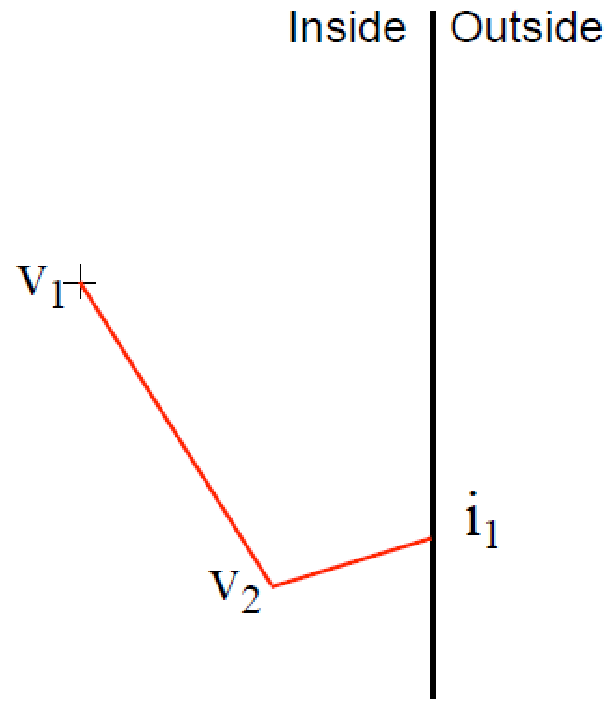


Current
Output

V_2, V_3

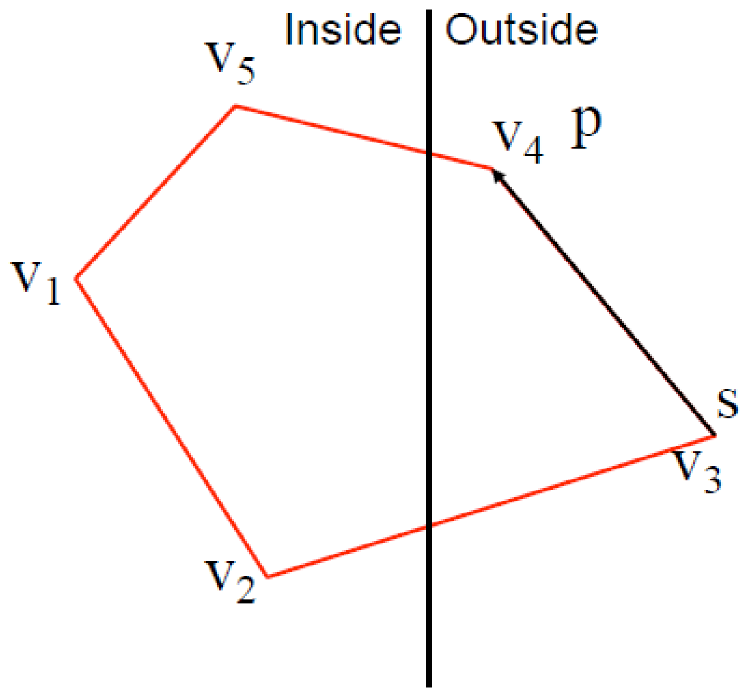


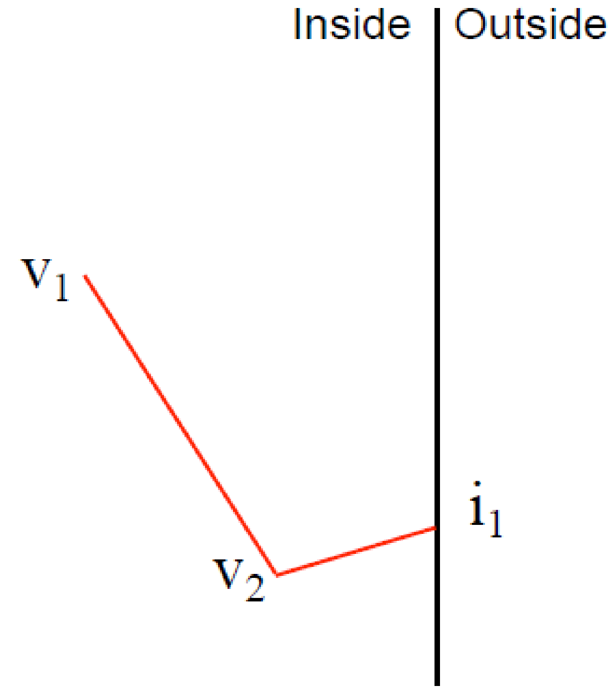
Inside, Outside
Output i_1



Current
Output

v_3, v_4

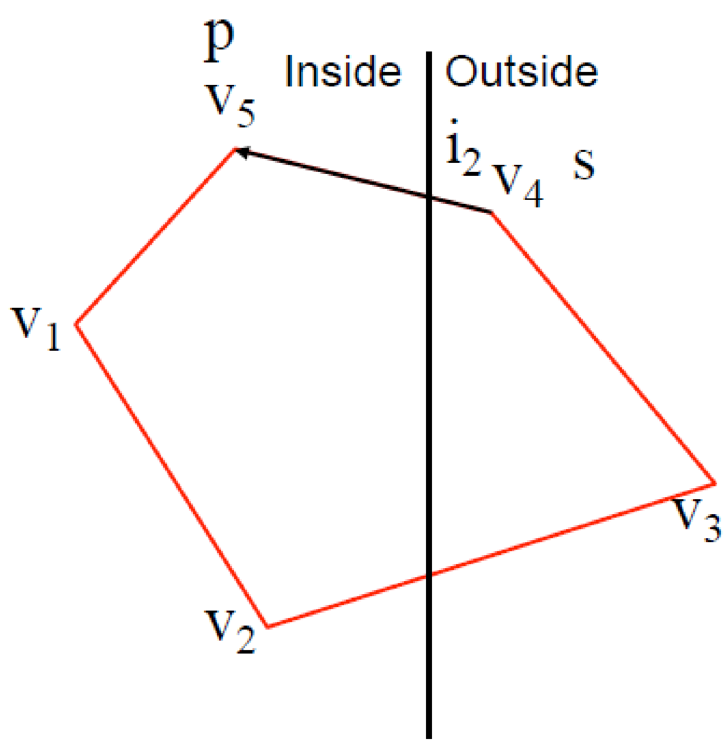




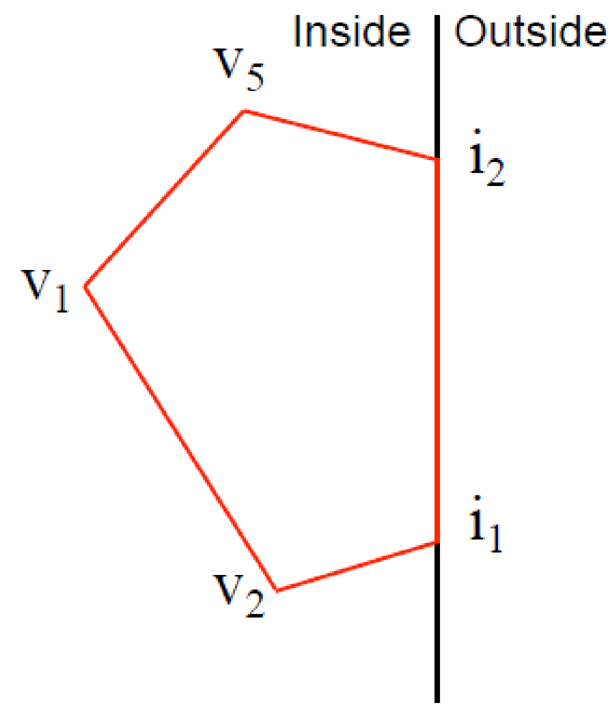
Outside, Outside
No output

Current
Output

v_4, v_5 – last edge…



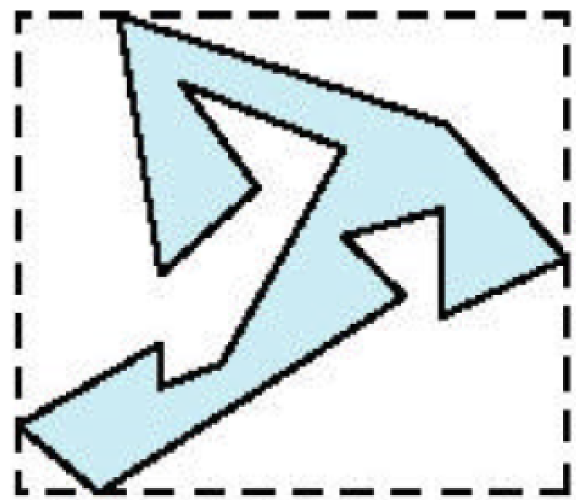
Outside, Inside
Output i_2, v_5



Current
Output

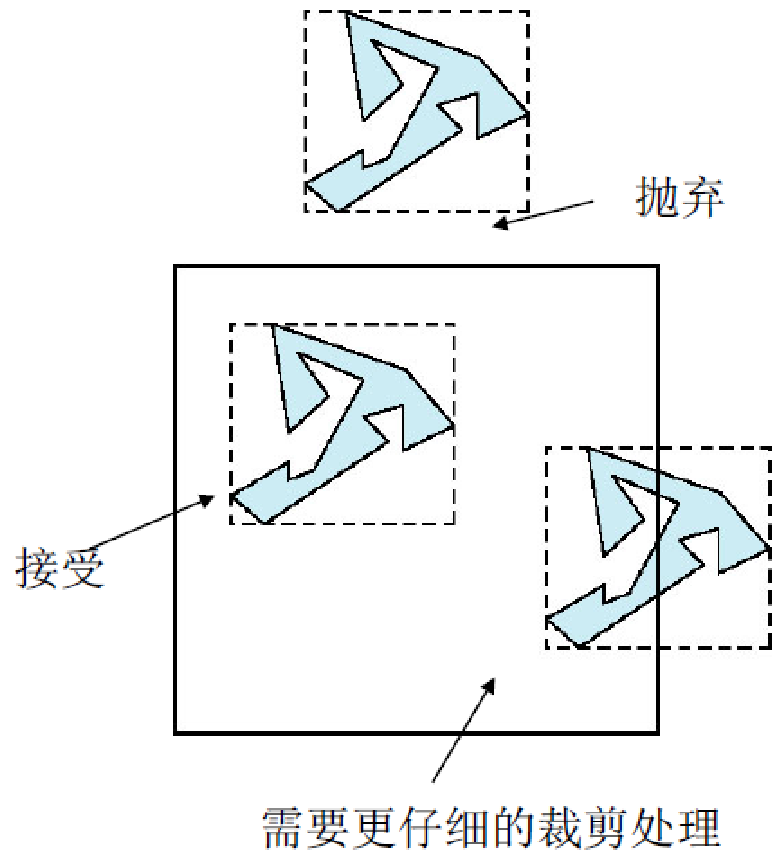
Bounding Box

- **不是直接对复杂多边形进行裁剪，而是先用一个方向与坐标轴平行的立方体或其它形状包围多边形**
  - 包围盒应尽可能得小
  - 容易计算出坐标的最大值与最小值



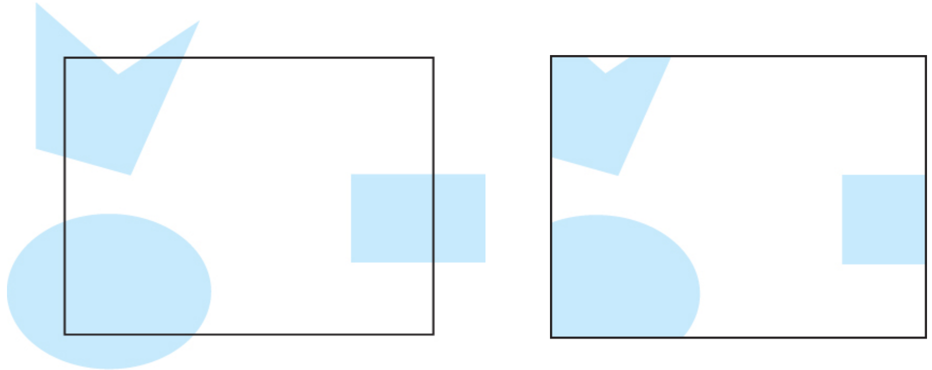
Bounding Box

- 通过直接基于包围盒确定多边形的接受与抛弃

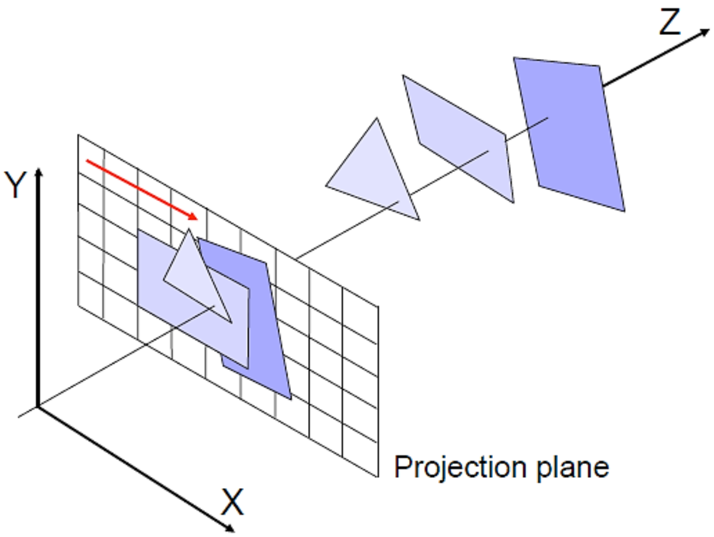


Outline

- Clipping



- Hidden Surface Removal



Hidden surface removal

- **Object Space Method**（对象空间）
  - ✓ a.k.a. Object Precision
  - ✓ Work in 3D before scan conversion
  - ✓ Usually independent of resolution
    - ➢ Important to maintain independence of output device(screen/ printer etc.)
  - ✓ Hidden Line/surface Remove

- **Image Space Method**（图像空间）
  - ✓ a.k.a. Image Precision
  - ✓ Work on per-pixel/per of fragment after scan conversion
  - ✓ Much faster, but resolution dependent
  - ✓ Z-Buffer/Depth Buffer

Framework of HSR in object space

```
for(each object in the world)  {
    determine those parts of the object whose view is
    unobstructed by other parts of it or any other object;
    draw those parts in the appropriate color;
}
```

Features

- High preciseness， independent of resolution of display devices (适合于精密的CAD工程领域)
- Complexity $O(n^2)$：
  - Each object should be compared with the other
  - n: object number
- Back surface culling,…

Framework of HSR in image space

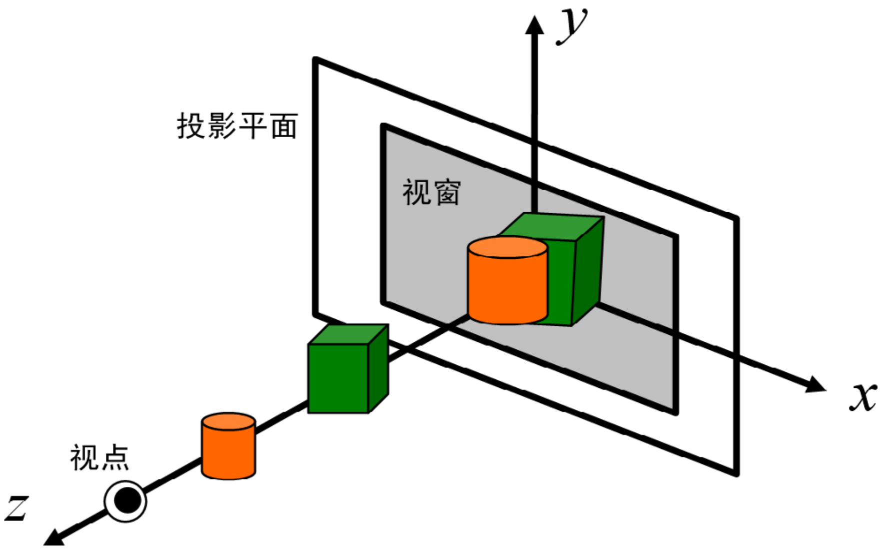
for(Each pixel in the image) {

connect **the pixel** and the viewpoint

find the nearest object；

compute the color for the pixel；

}



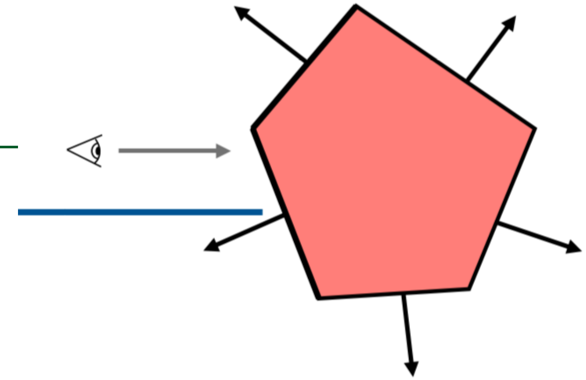
Features

- The image is constrained by resolution of the display devices
- Complexity O(nN)：
  - Objects should be sorted for each pixel (use coherence!每个象素都需对物体排序)
  - n: the number of primitives (polygons)
  - N: the number of pixels
- Algorithms： z-buffer

Object Space Method

- Determine visibility on object or polygon level
  - Using camera coordinates
- Resolution independent
  - Explicitly compute visible portions of polygons
- Early in pipeline
  - After clipping
- Requires depth-sorting
  - Painter's algorithm
  - BSP trees

Back face culling

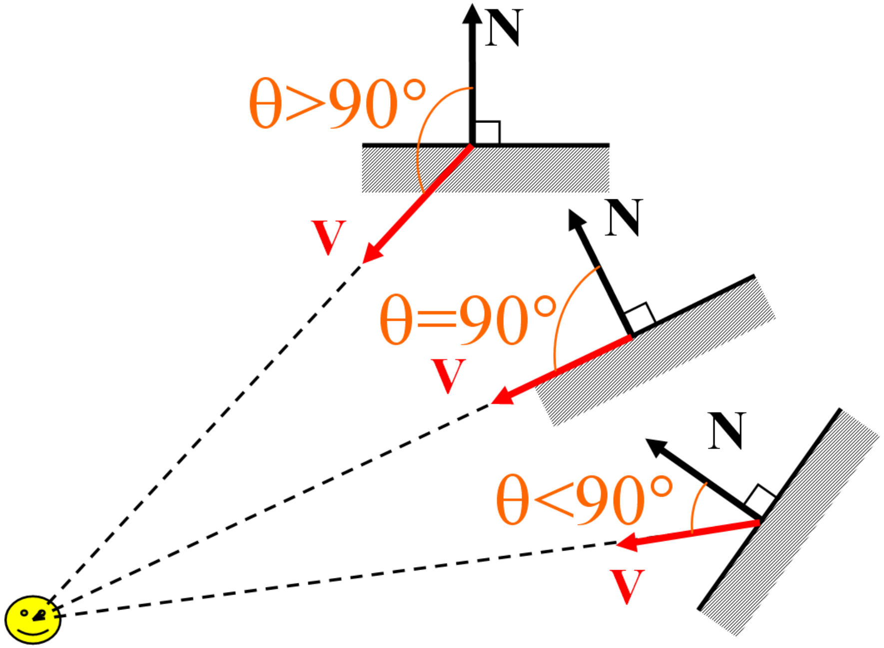


- In a closed polygonal surface
  - i.e. the surface of a polyhedral volume or a solid polyhedron
  - The faces whose outward normals point away from the viewer are not visible
  - Such back-facing faces can be eliminated from further processing

- Elimination of back-faces is called back-face culling

Back face culling

- Let V be the viewing direction from the object to the camera; n the normal of the face to be tested
  - **N**·**V**<0： invisible
  - **N**·**V**≥0: visible



Back face culling

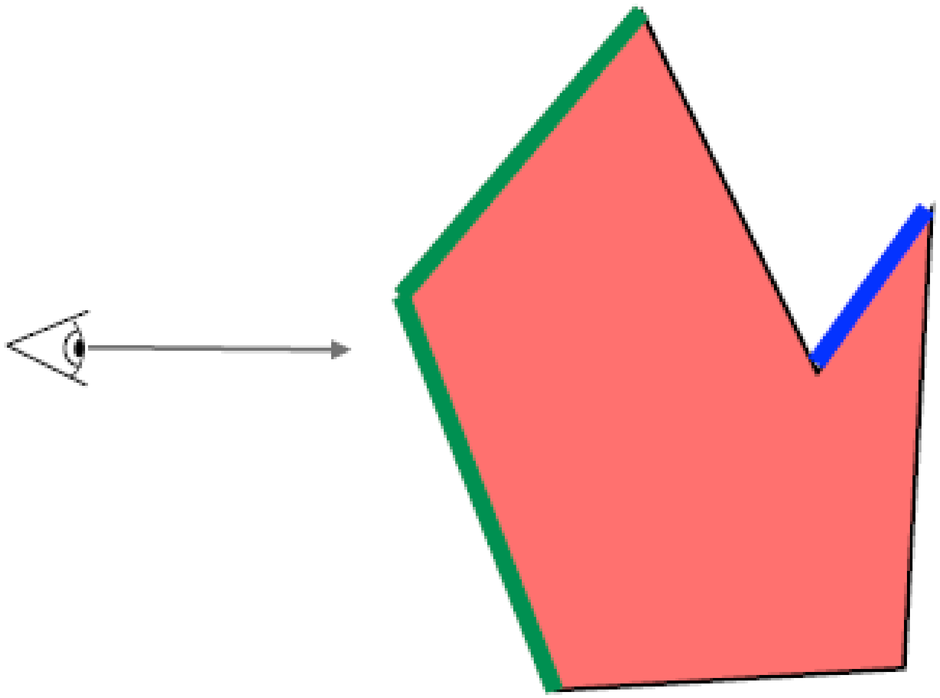
- Determine back & front faces using sign of inner product nv

$$n \cdot v = n_x v_x + n_y v_y + n_z v_z = \|n\| \cdot \|v\| \cos\theta$$

- In a convex object :
  - Invisible back faces
  - All front faces entirely visible $\Rightarrow$ solves hidden surfaces problem
- In non-convex object:
  - Invisible back faces
  - Front faces can be visible, invisible, or partially visible

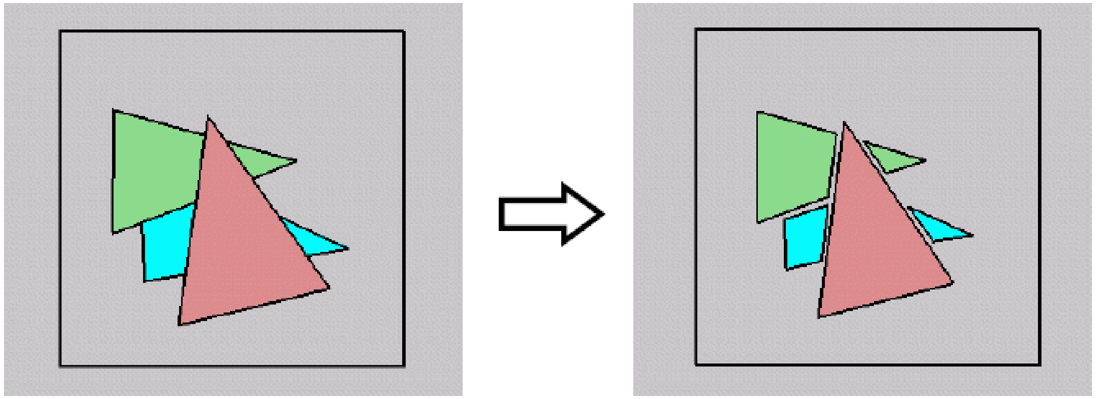
Limitations

- Only applicable to convex polyhedra

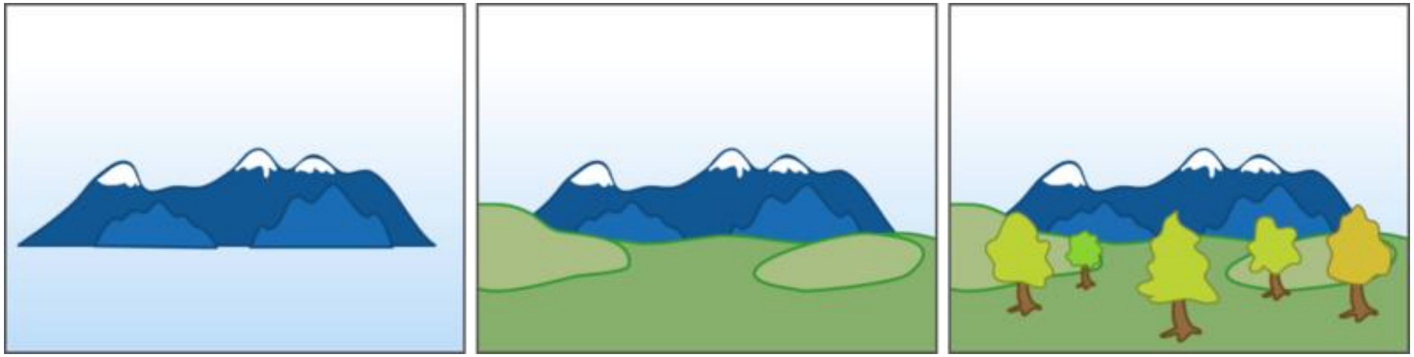


Painter's Algorithm

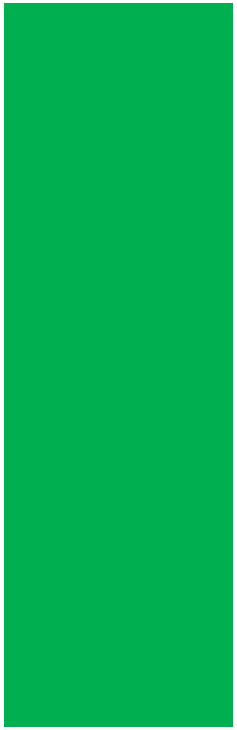
- Simple: render the polygons from back to front, "painting over" previous polygons



- Draw cyan, then green, then red
- Will this work in general?

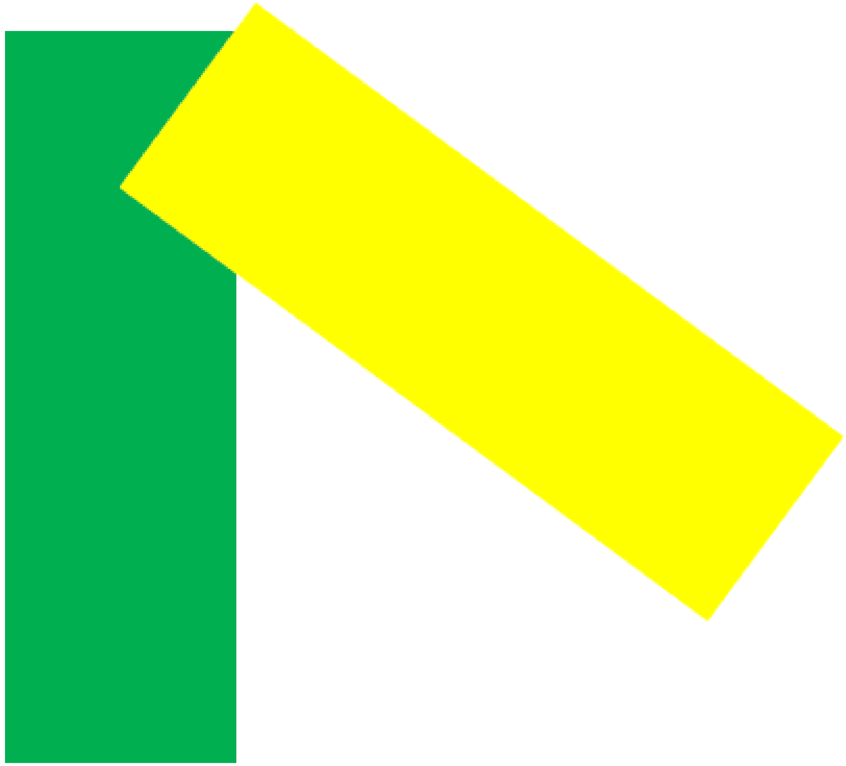


For 2D application



Draw items one at a time

For 2D application



Draw items one at a time

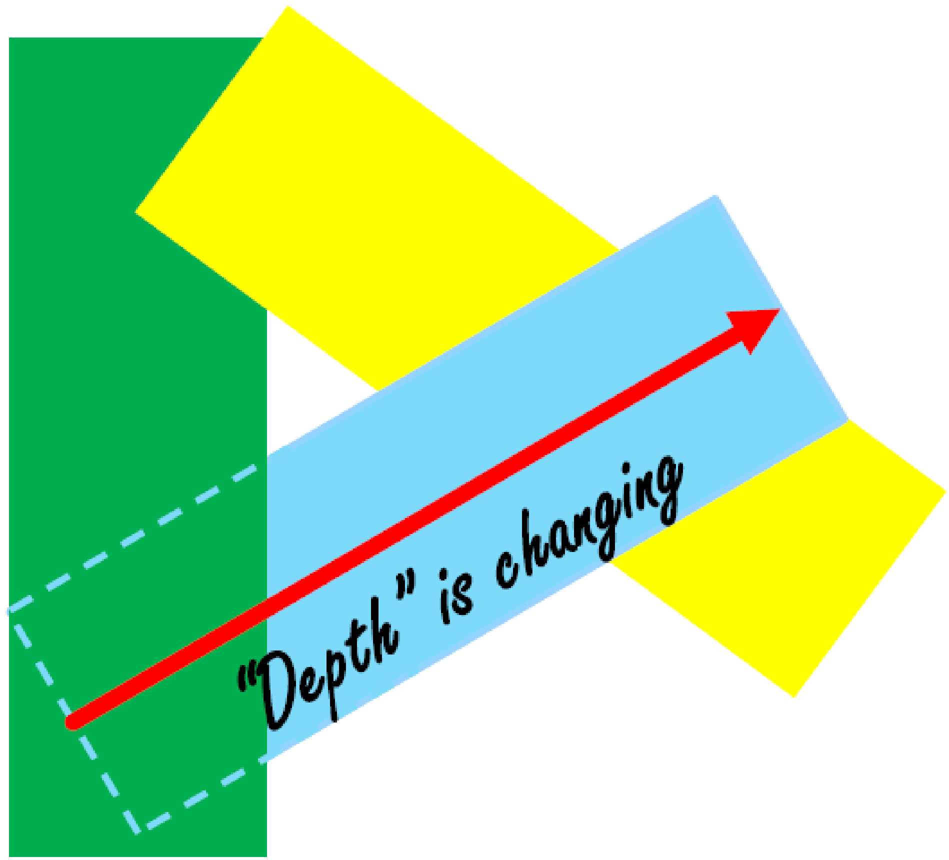
For 2D application

Draw items one at a time

Painter's Algorithm:Problem

What Order?

Painter’s Algorithm:Problem



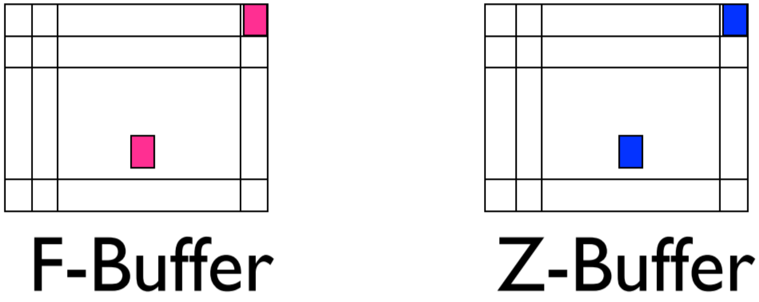
What Order?

Painter's Algorithm:Problem

- Intersecting polygons present a problem
- Even non-intersecting polygons can form a cycle with no valid visibility order:

Z-buffer algorithm

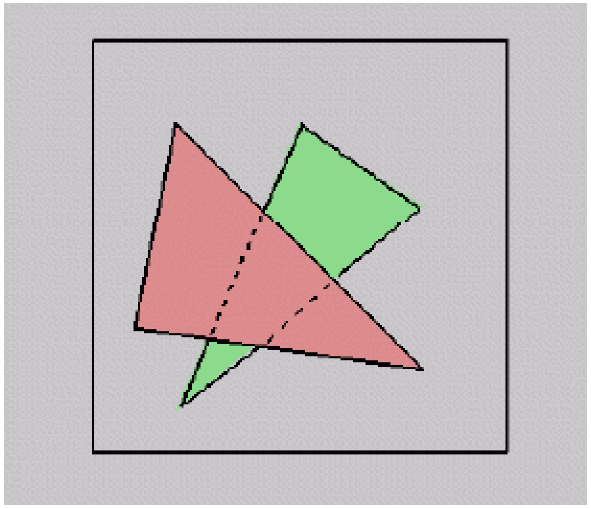
- Image precision algorithm
  - Apart from a frame buffer F in which color values are stored,
  - it also needs a z-buffer, of the same size as the frame buffer, to store depth (z) values

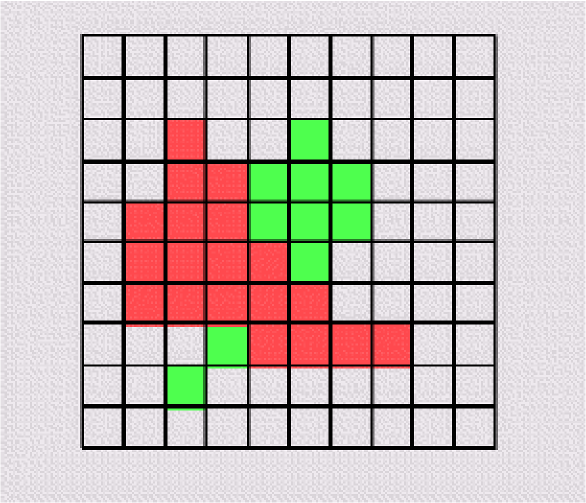


A.K.A. depth-buffer method

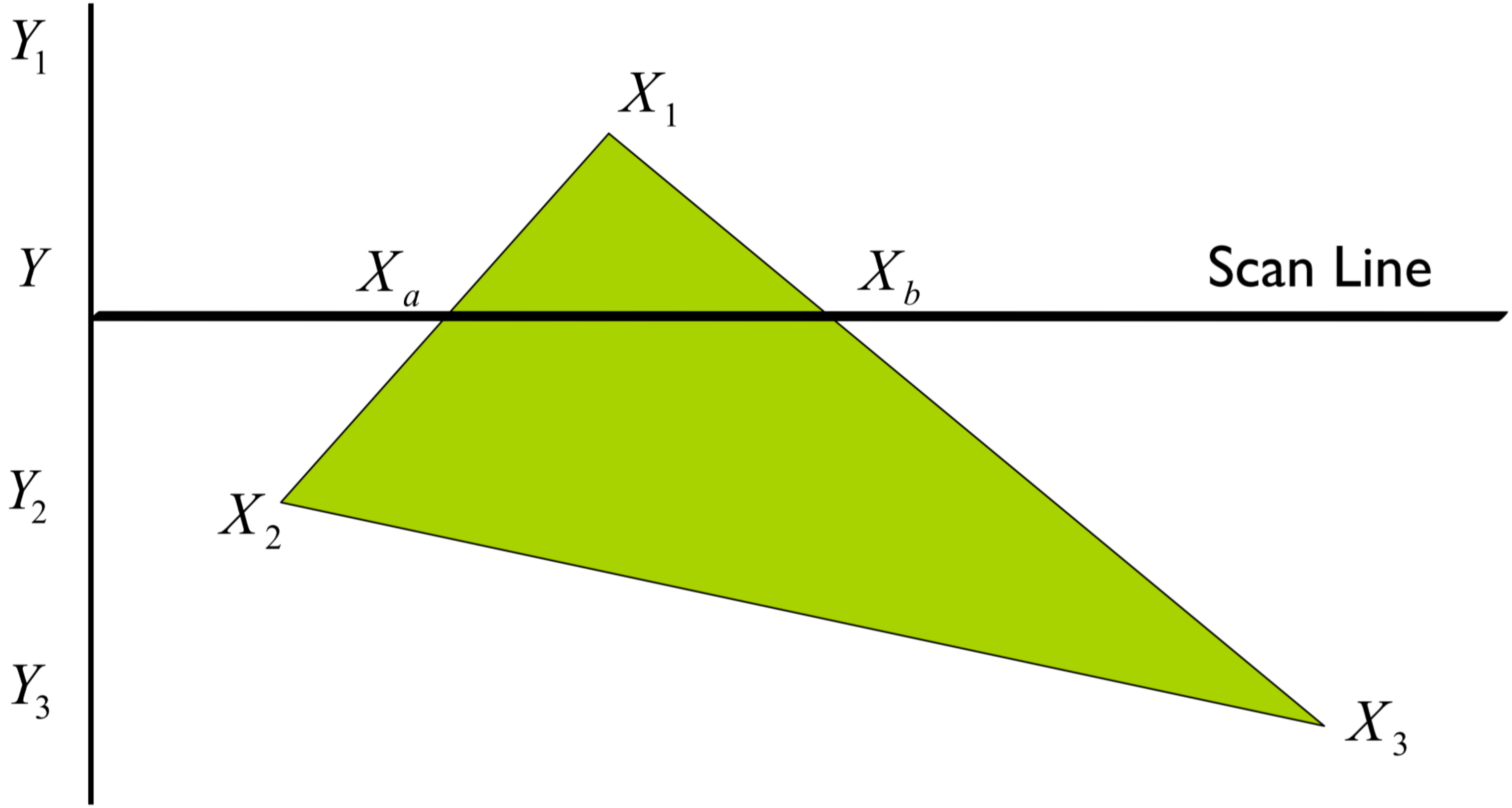
Z-buffer algorithm

- What happens if multiple primitives occupy the same pixel on the screen?
- Which is allowed to paint the pixel?

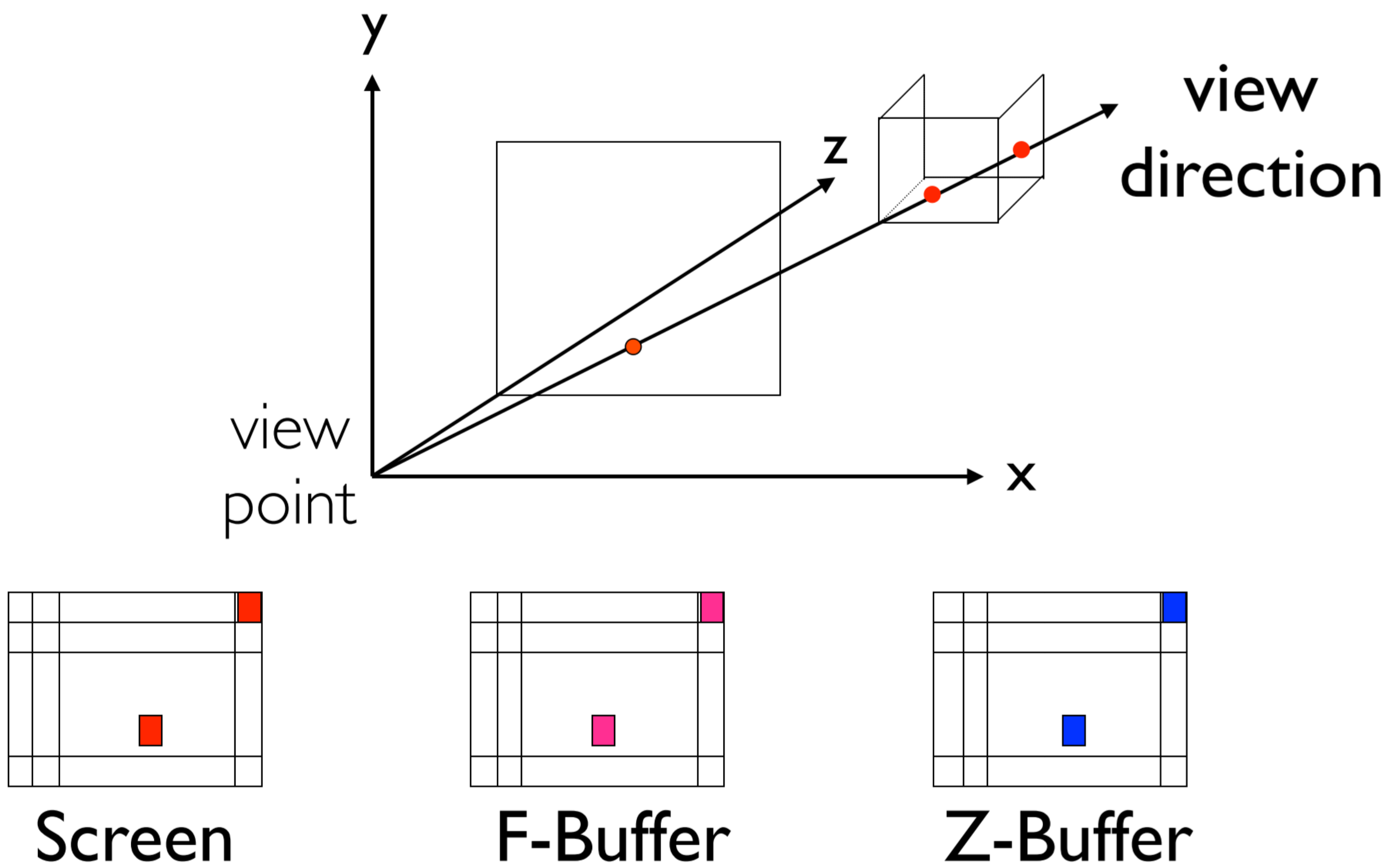




Polygon Scan Conversation



Z-buffer algorithm



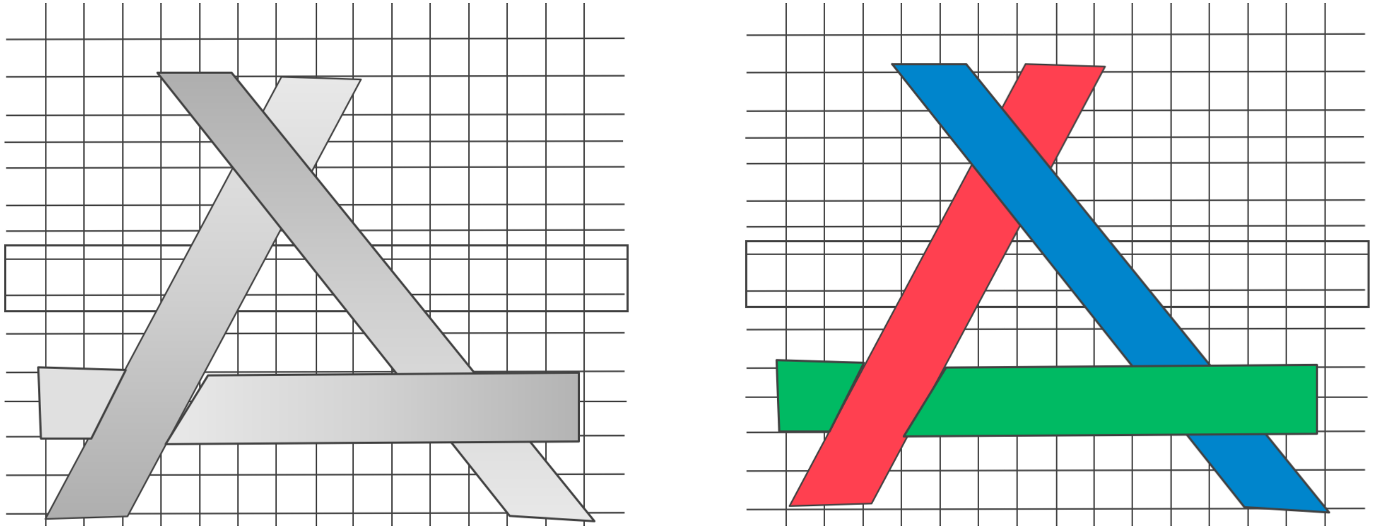
Z-Buffer Pseudo-code

- for (j=0; j<SCREEN_HEIGHT; j++)
  - for (i=0; i<SCREEN_WIDTH; i++) {
    - WriteToFrameBuffer(i, j, BackgroundColor);
    - WriteToZBuffer(i, j, MAX);
  - }

- for (each polygon)
  - for (each pixel in polygon's projection) {
    - z = polygon's z value at (i, j) ;
    - if (z < ReadFromZBuffer(i, j)) {
      - WriteToFrameBuffer(i, j, polygon's color at (i, j));
      - WriteToZBuffer(i, j, z);
    - }
  - }

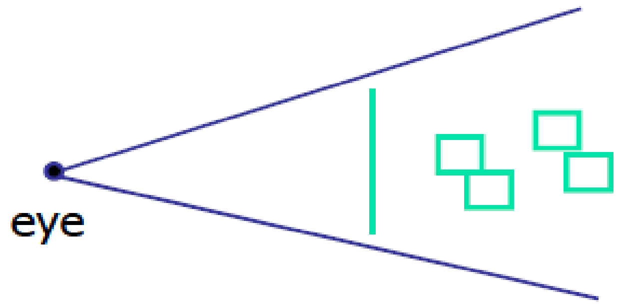
Z-Buffer Pros

- Simple!!!
- Easy to implement in hardware
  - Hardware support in all graphics cards today
- Polygons can be processed in arbitrary order
- Easily handles polygon interpenetration



Z-Buffer cons

- Poor for scenes with high depth complexity
  - Need to render all polygons, even if most are invisible

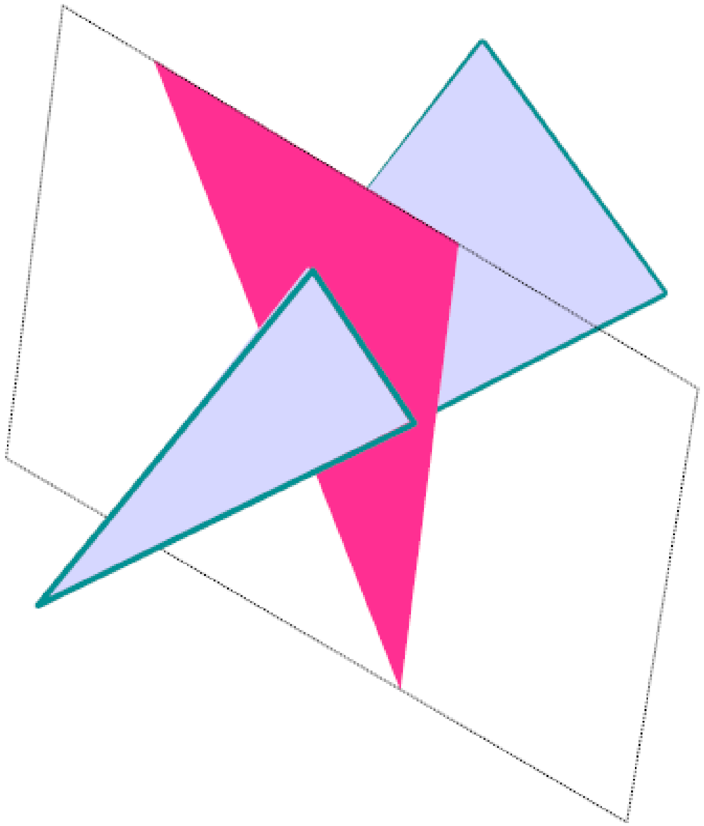


- Shared edges/overlaps handled inconsistently
  - *Ordering dependent*

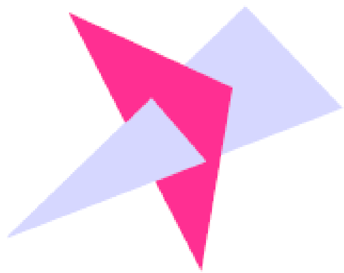
Binary Space Partitioning Trees

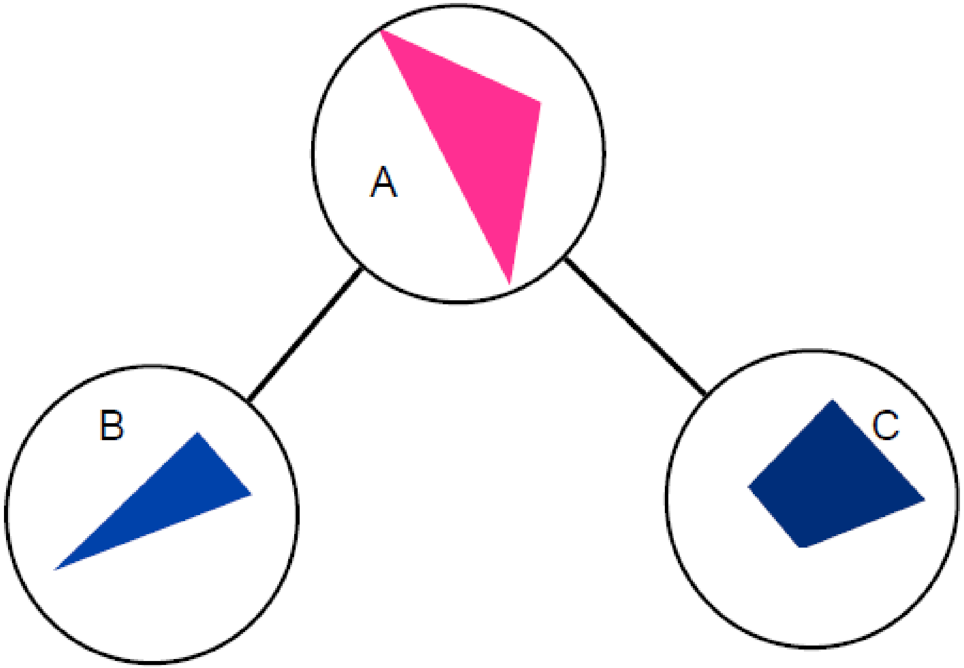
- BSP Tree
  - Very efficient for a static group of 3D polygons as seen from an arbitrary viewpoint
  - Correct order for Painter's algorithm is determined by a suitable traversal of the binary tree of polygons

BSP Tree



BSP Tree





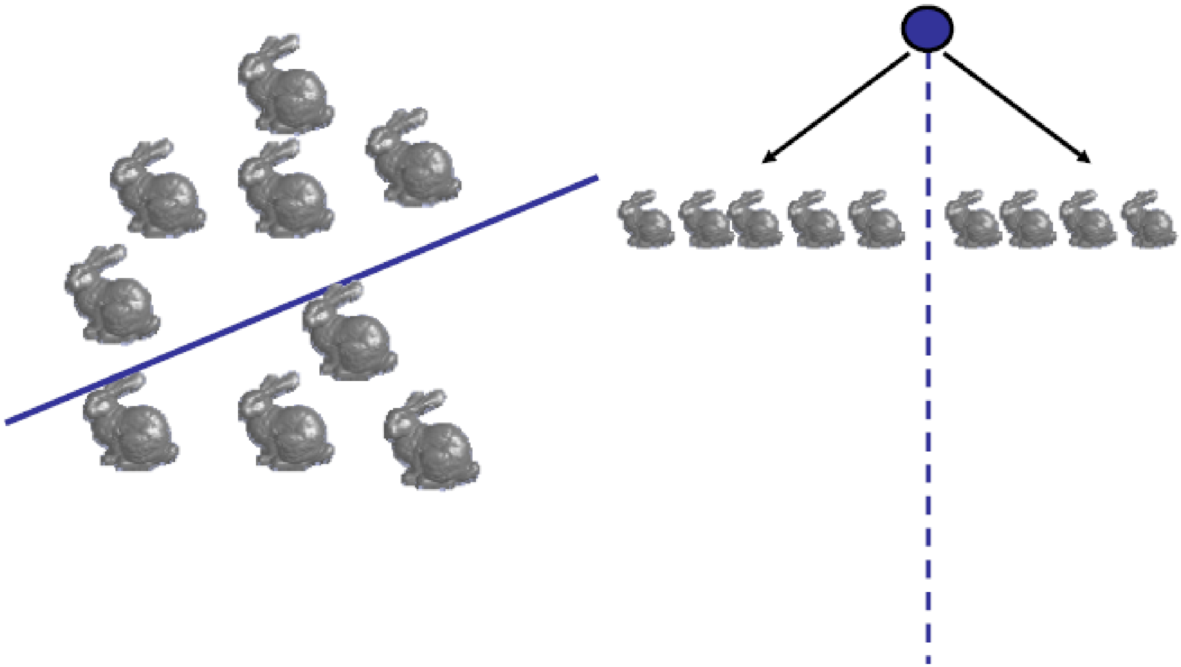
Binary Space Partition Trees

- BSP Tree: partition space with binary tree of planes
- Idea: divide space recursively into half-spaces by choosing splitting planes that separate objects in scene

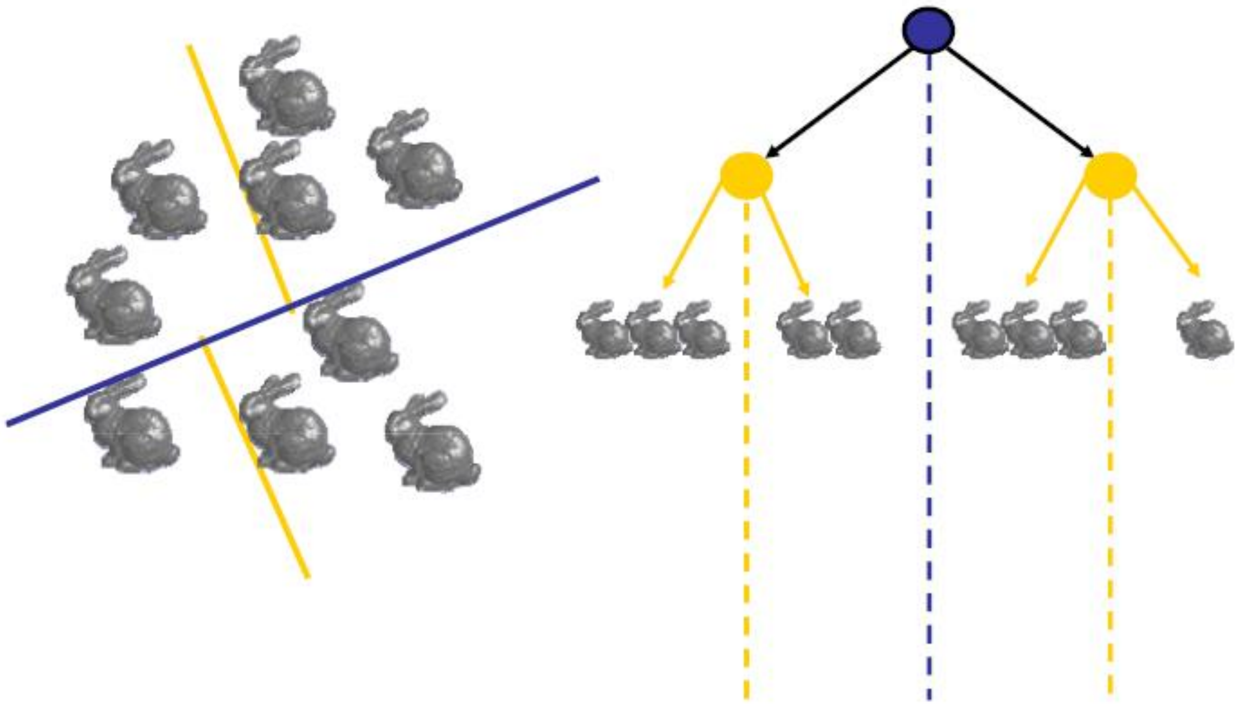
- Preprocessing: create binary tree of planes
- Runtime: correctly traversing this tree enumerates objects from back to front

Creating BSP Trees: Objects

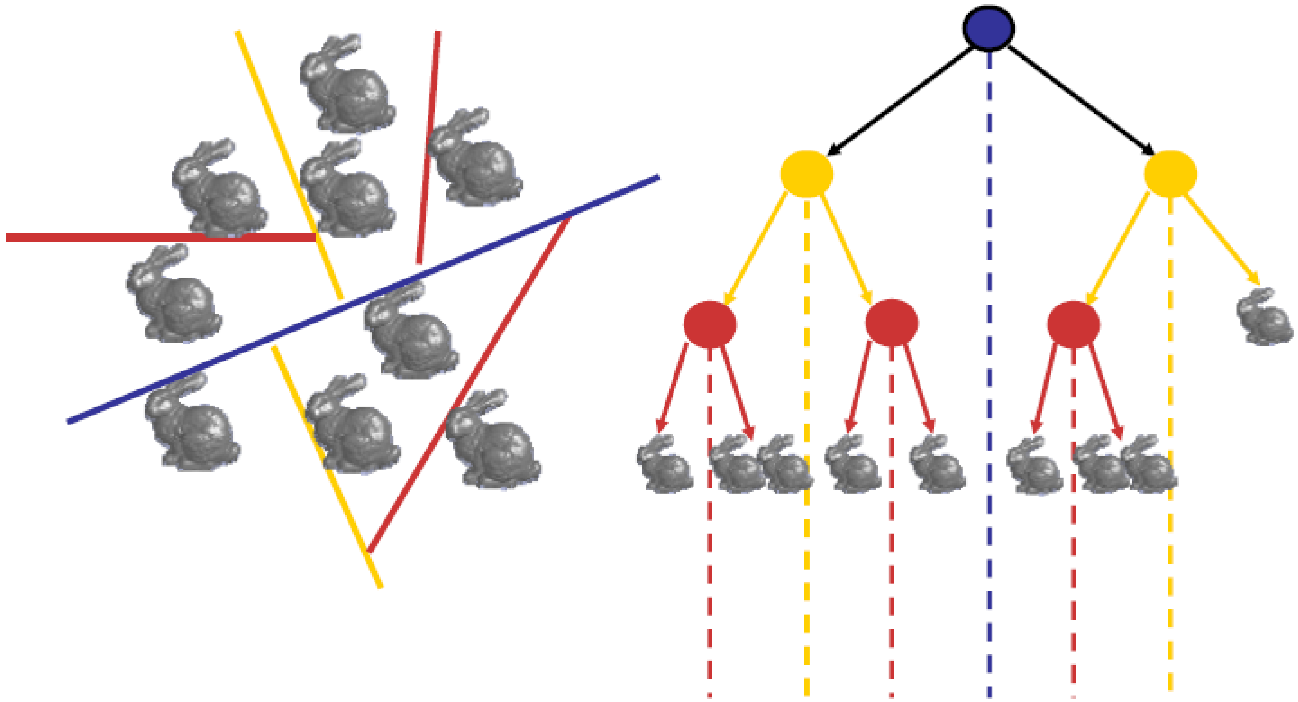
Creating BSP Trees: Objects



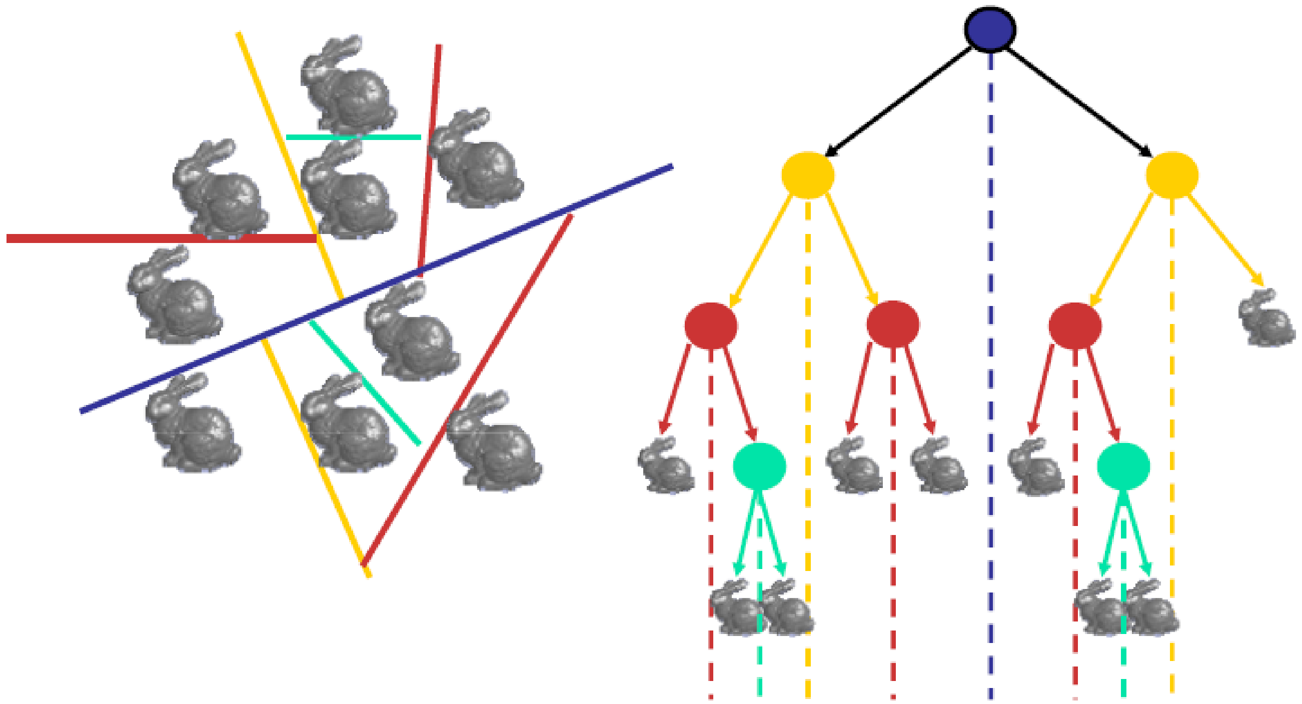
Creating BSP Trees: Objects



Creating BSP Trees: Objects

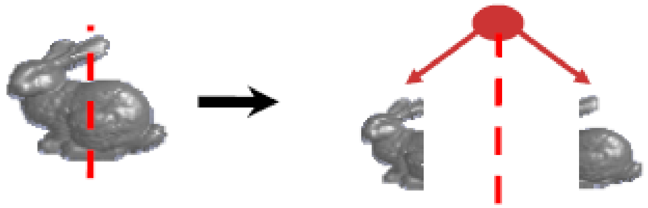


Creating BSP Trees: Objects



Splitting Objects

- No bunnies were harmed in previous example
- But what if a splitting plane passes through an object?
  - Split the object; give half to each node



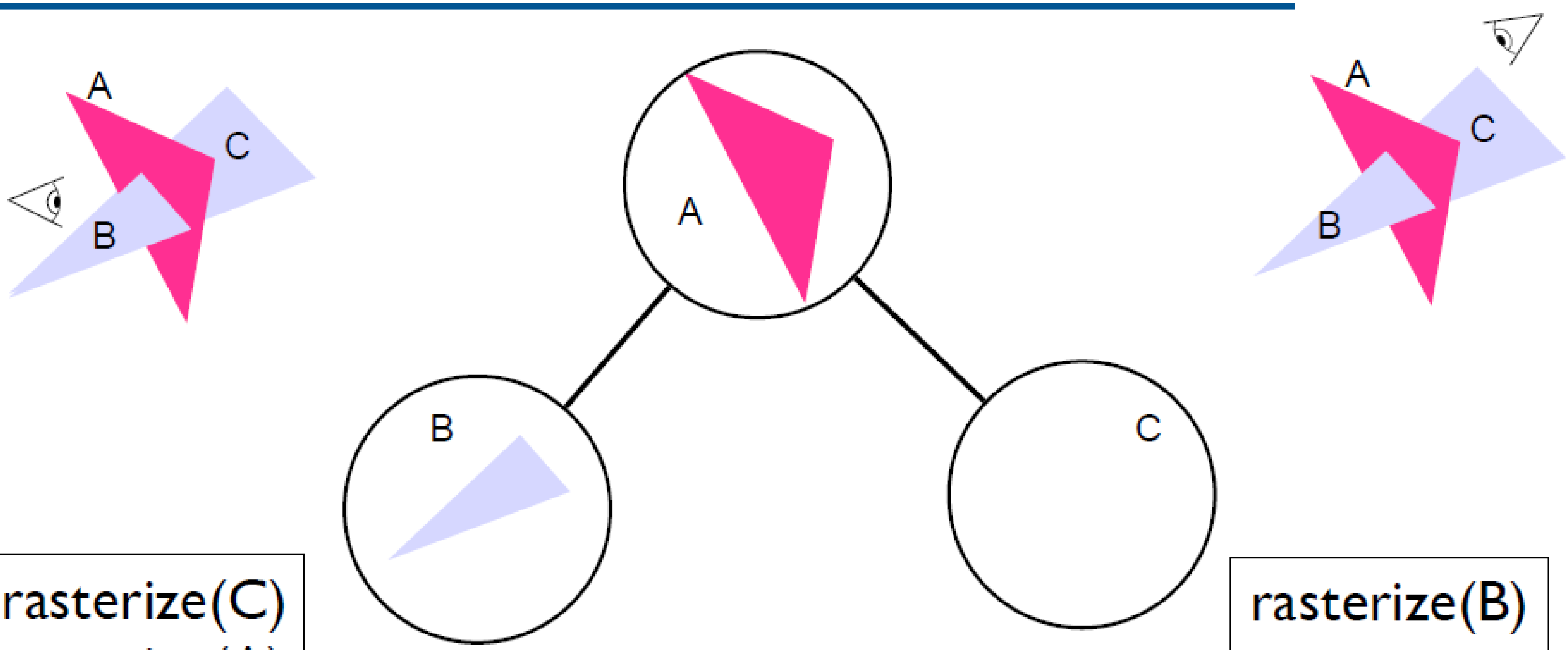
Traversing BSP-Trees

- Tree creation independent of viewpoint
  - Preprocessing step
- Tree traversal uses viewpoint
  - Runtime, happens for many different viewpoints

Traversing BSP-Trees

- Each plane divides world into near and far
  - For given viewpoint, decide which side is near and which is far
    - Check which side of plane viewpoint is on independently for each tree vertex
    - Tree traversal differs depending on viewpoint!
  - Recursive algorithm
    - Recurse on far side
    - Draw object
    - Recurse on near side

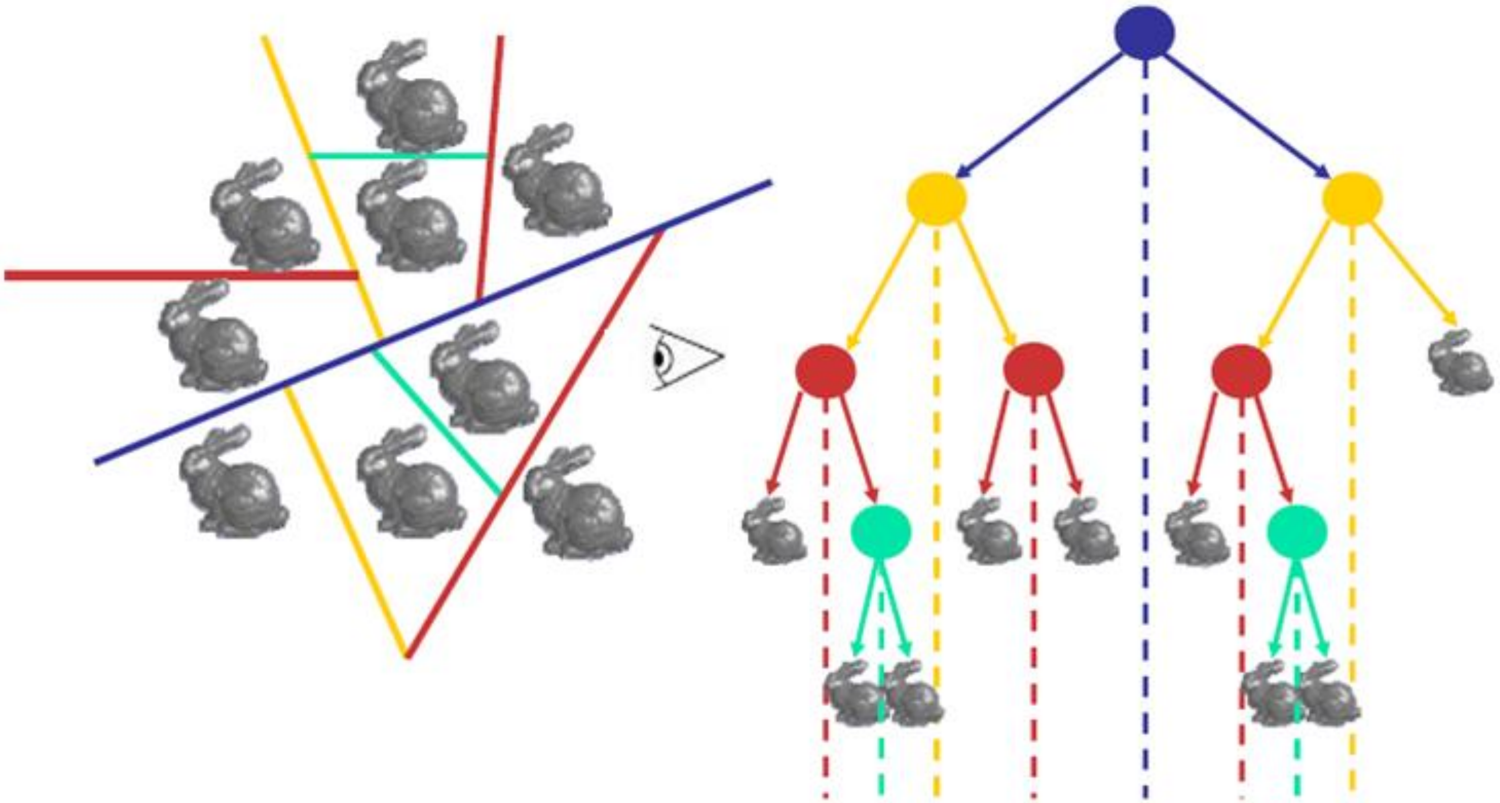
Traversing BSP-Trees



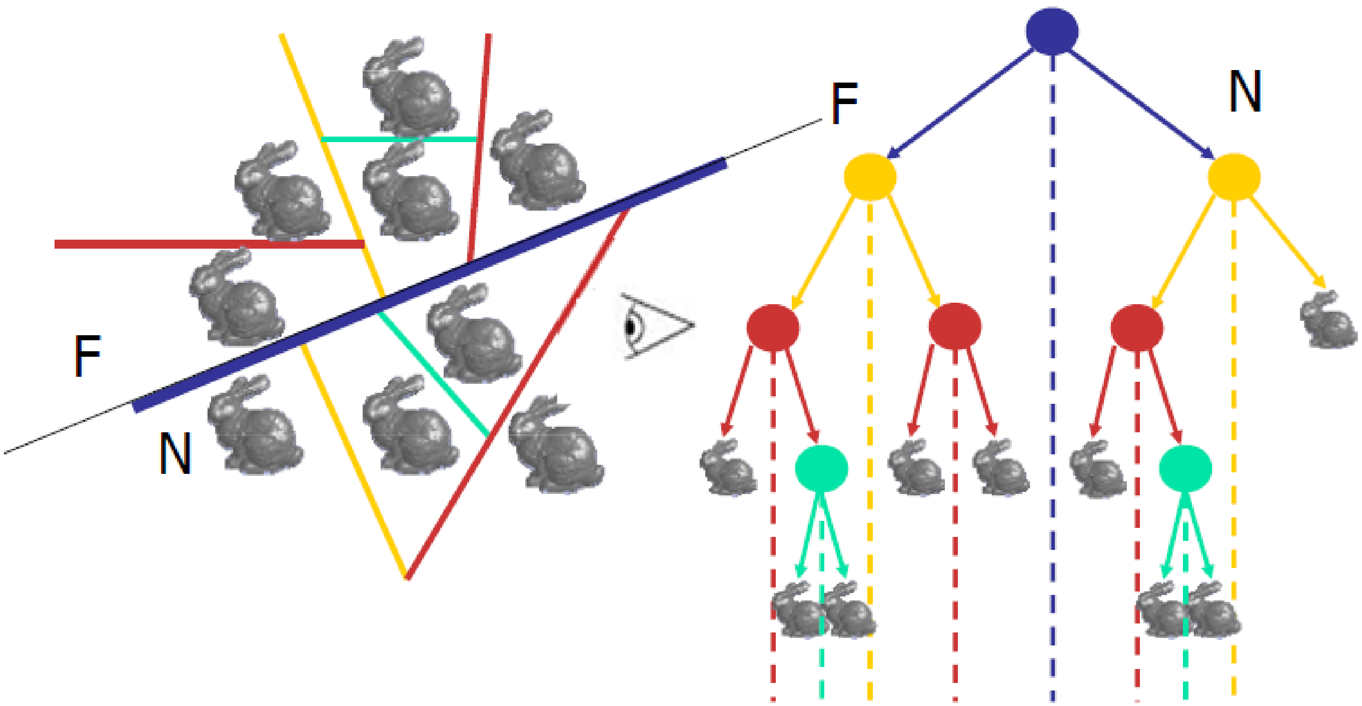
rasterize(C)
rasterize(A)
rasterize(B)

rasterize(B)
rasterize(A)
rasterize(C)

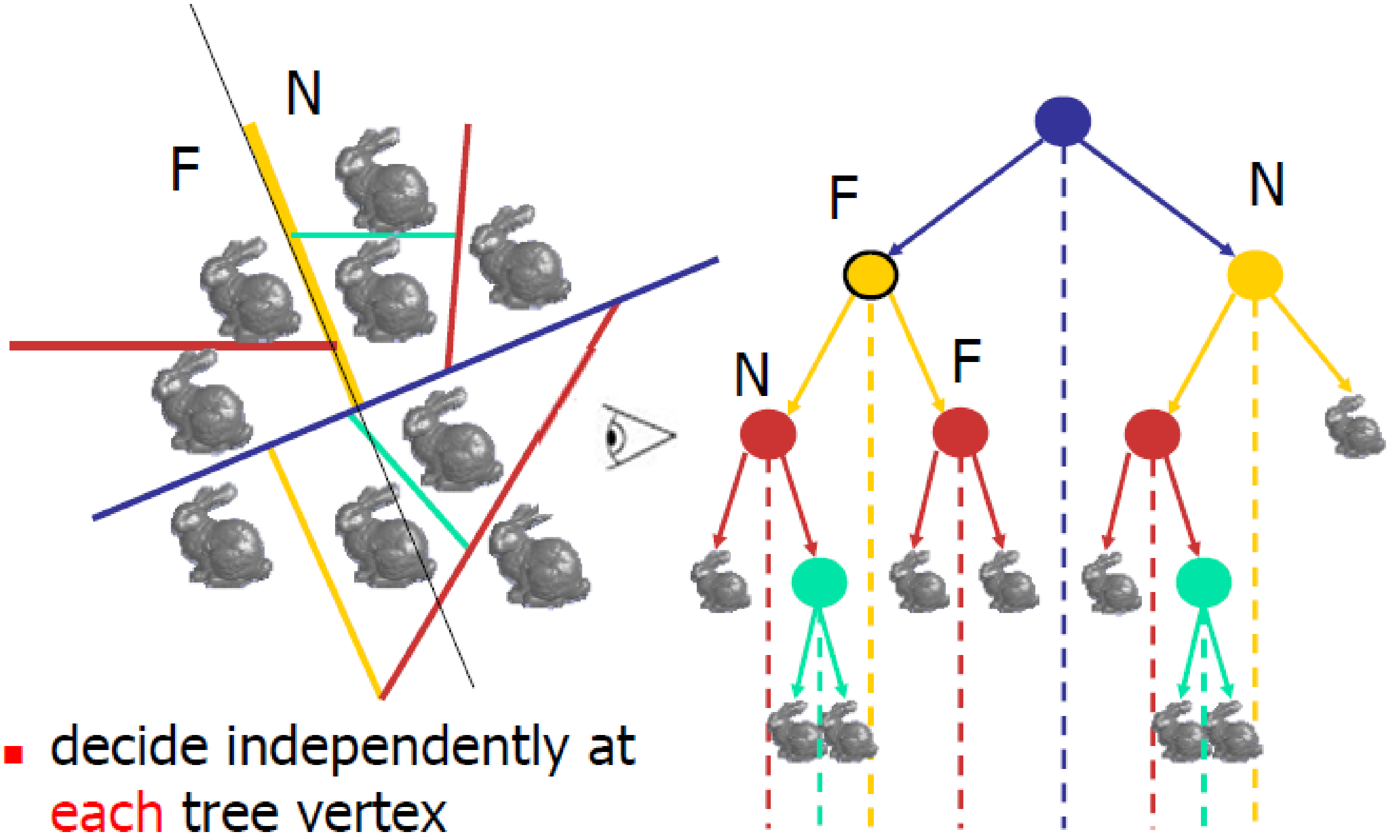
BSP-Trees: Viewpoint A



BSP-Trees: Viewpoint A

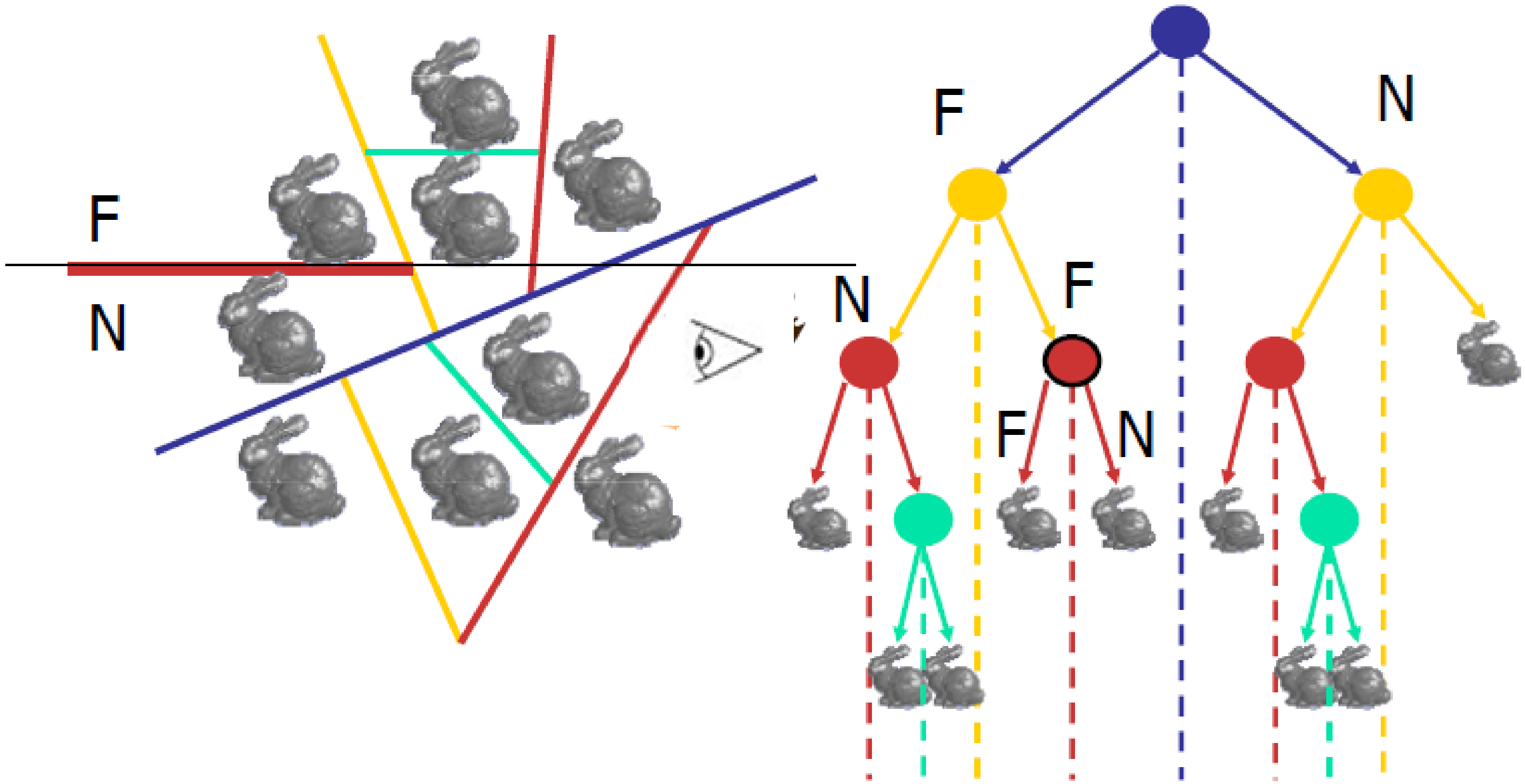


BSP-Trees: Viewpoint A

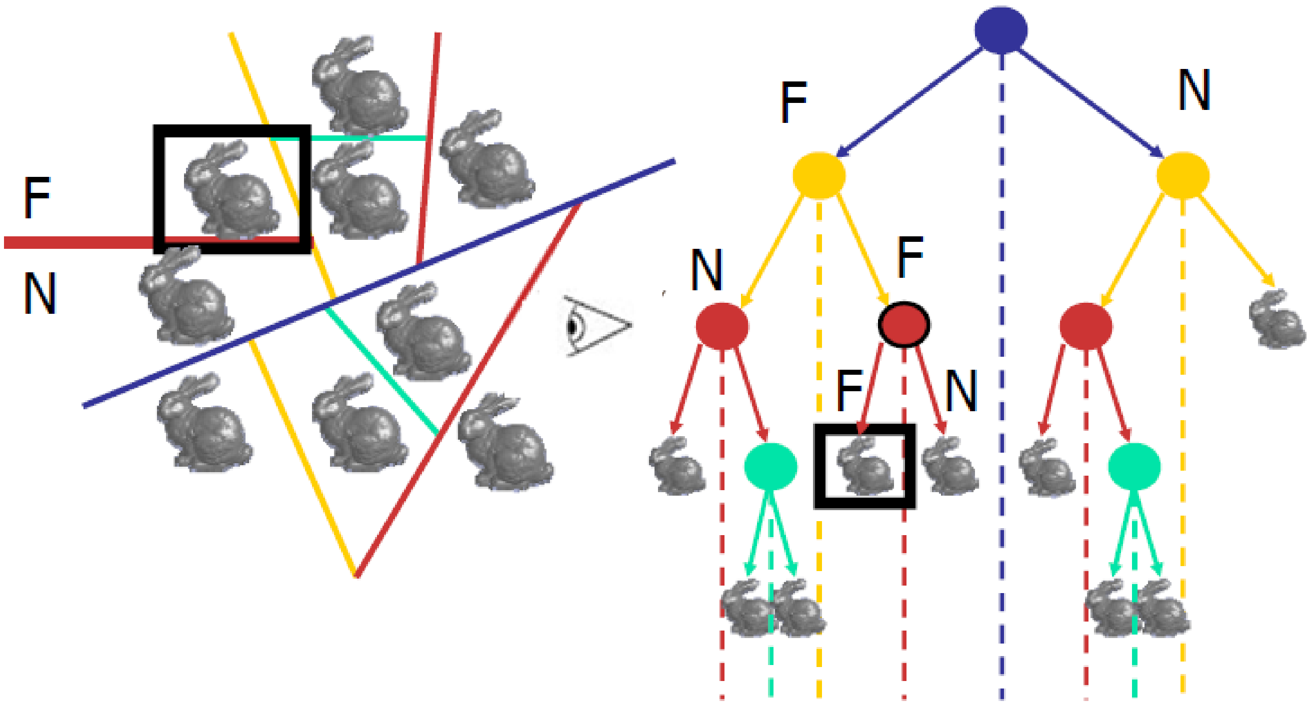


- decide independently at each tree vertex
- not just left or right child!

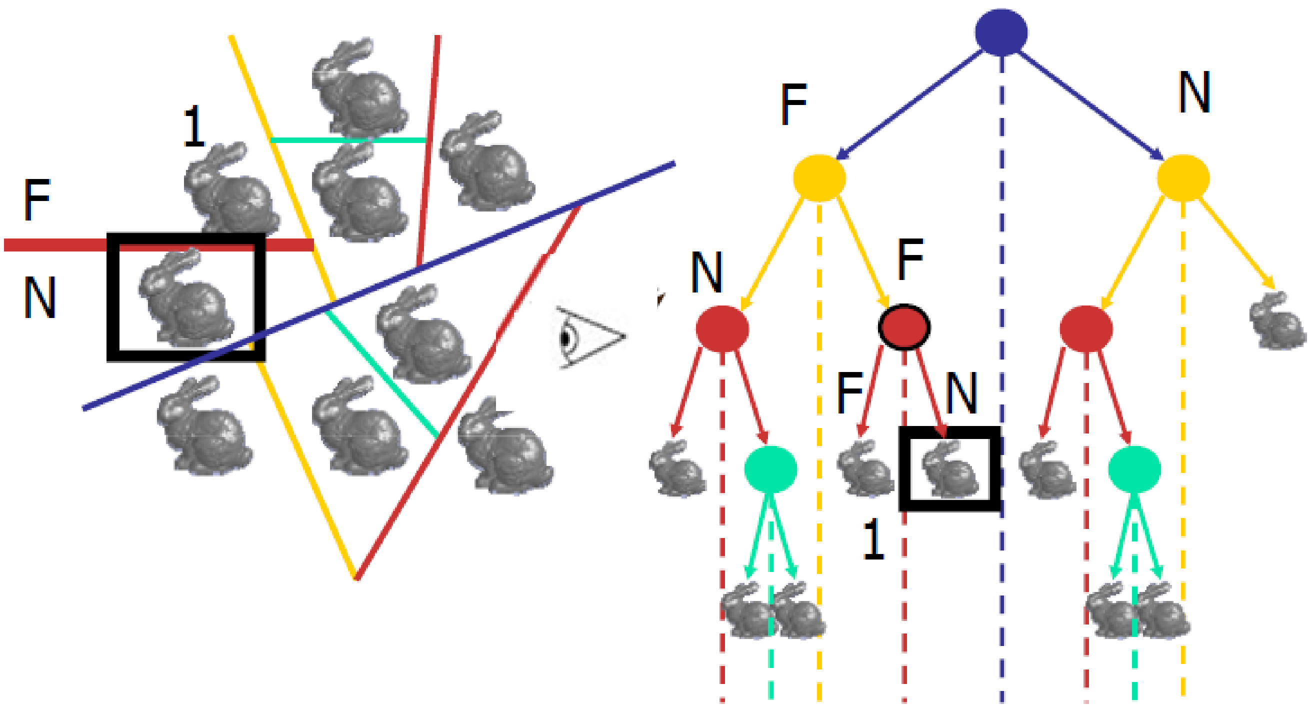
BSP-Trees: Viewpoint A



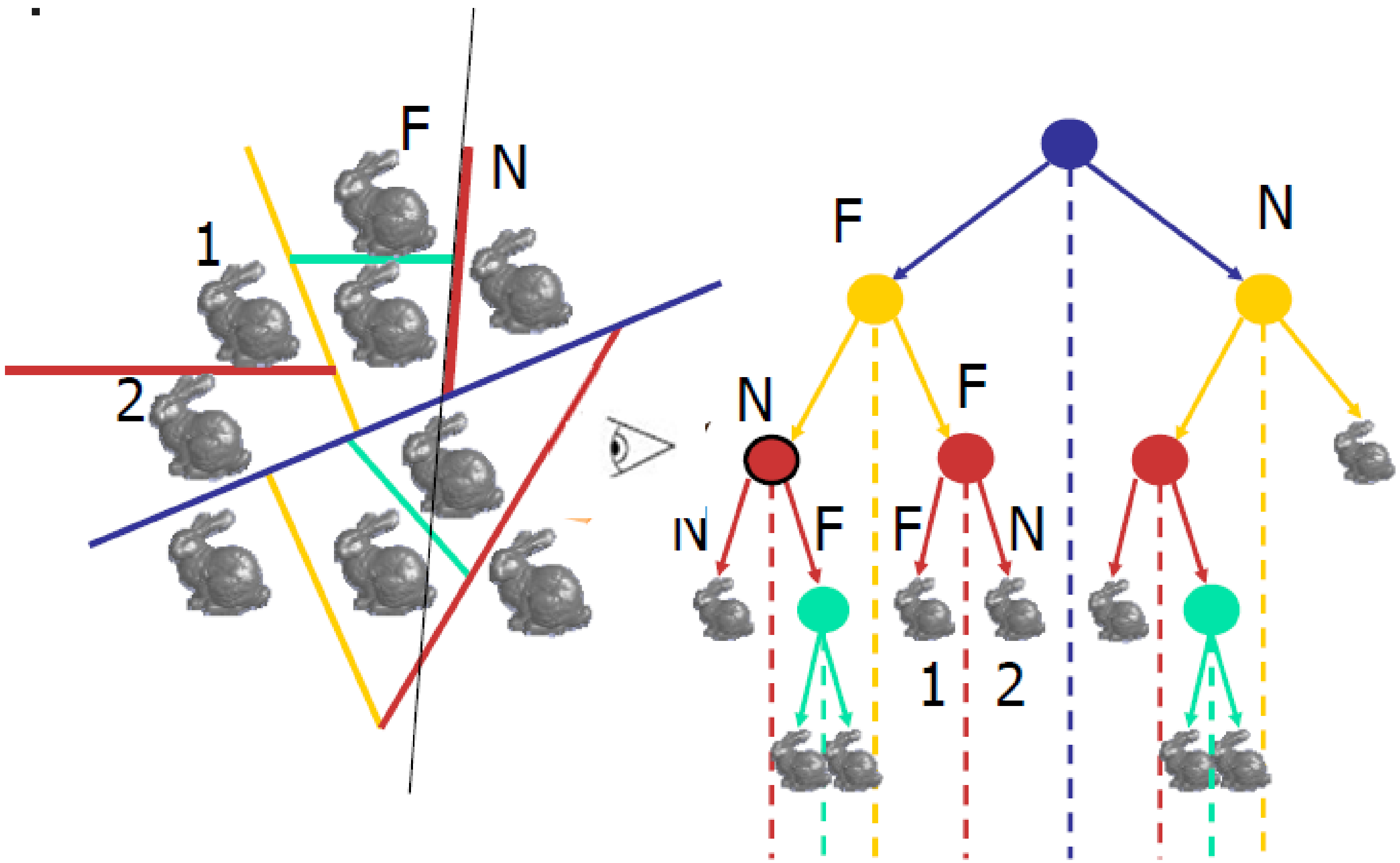
BSP-Trees: Viewpoint A



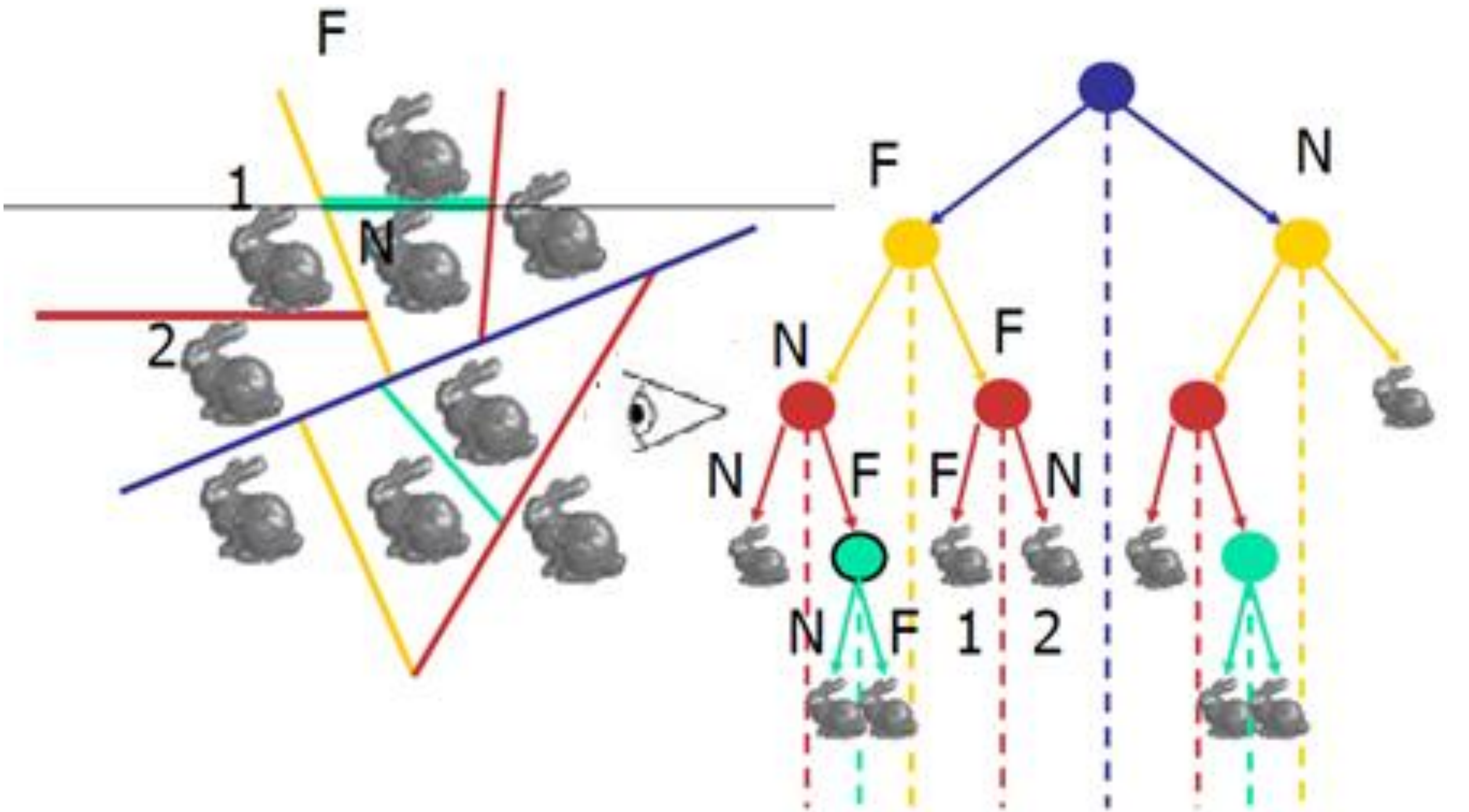
BSP-Trees: Viewpoint A



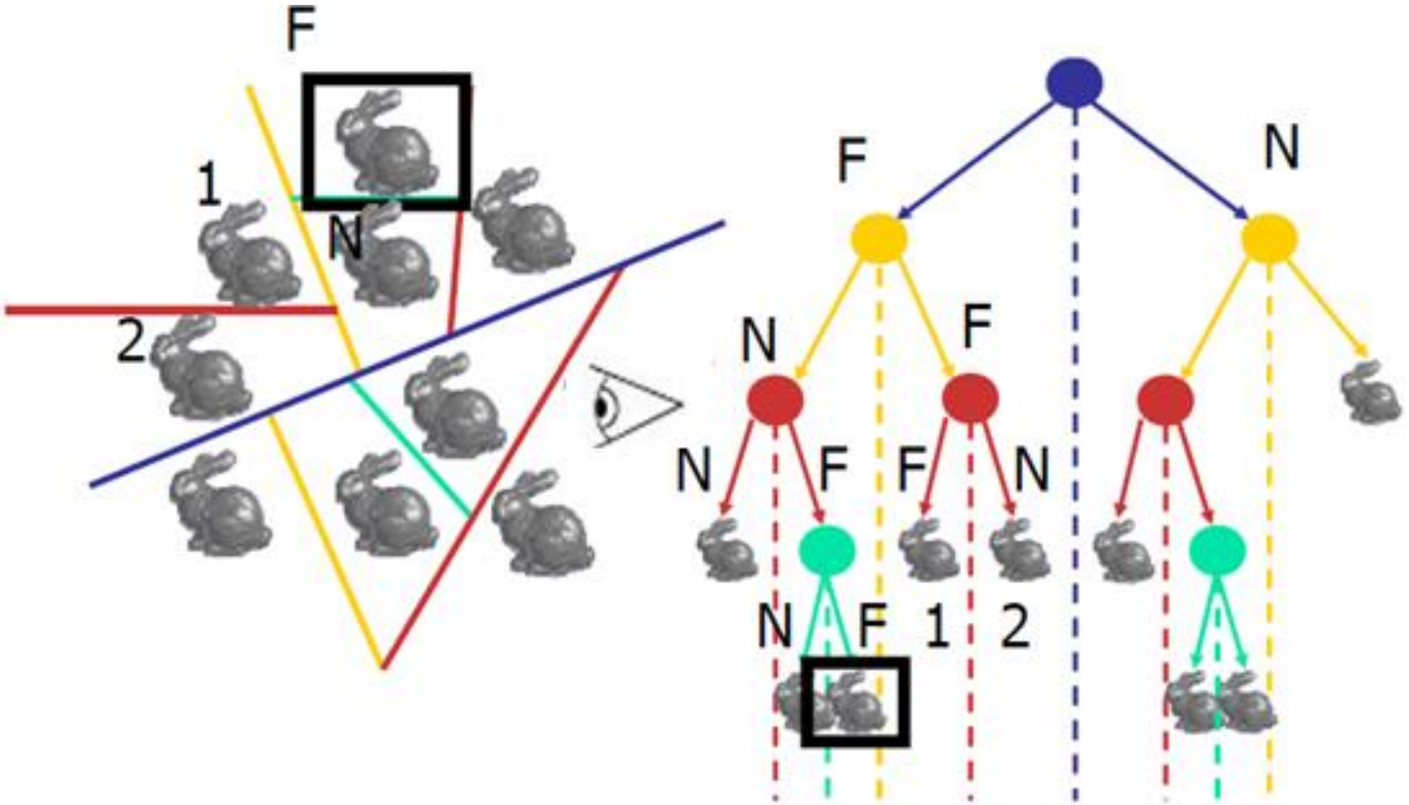
BSP-Trees: Viewpoint A



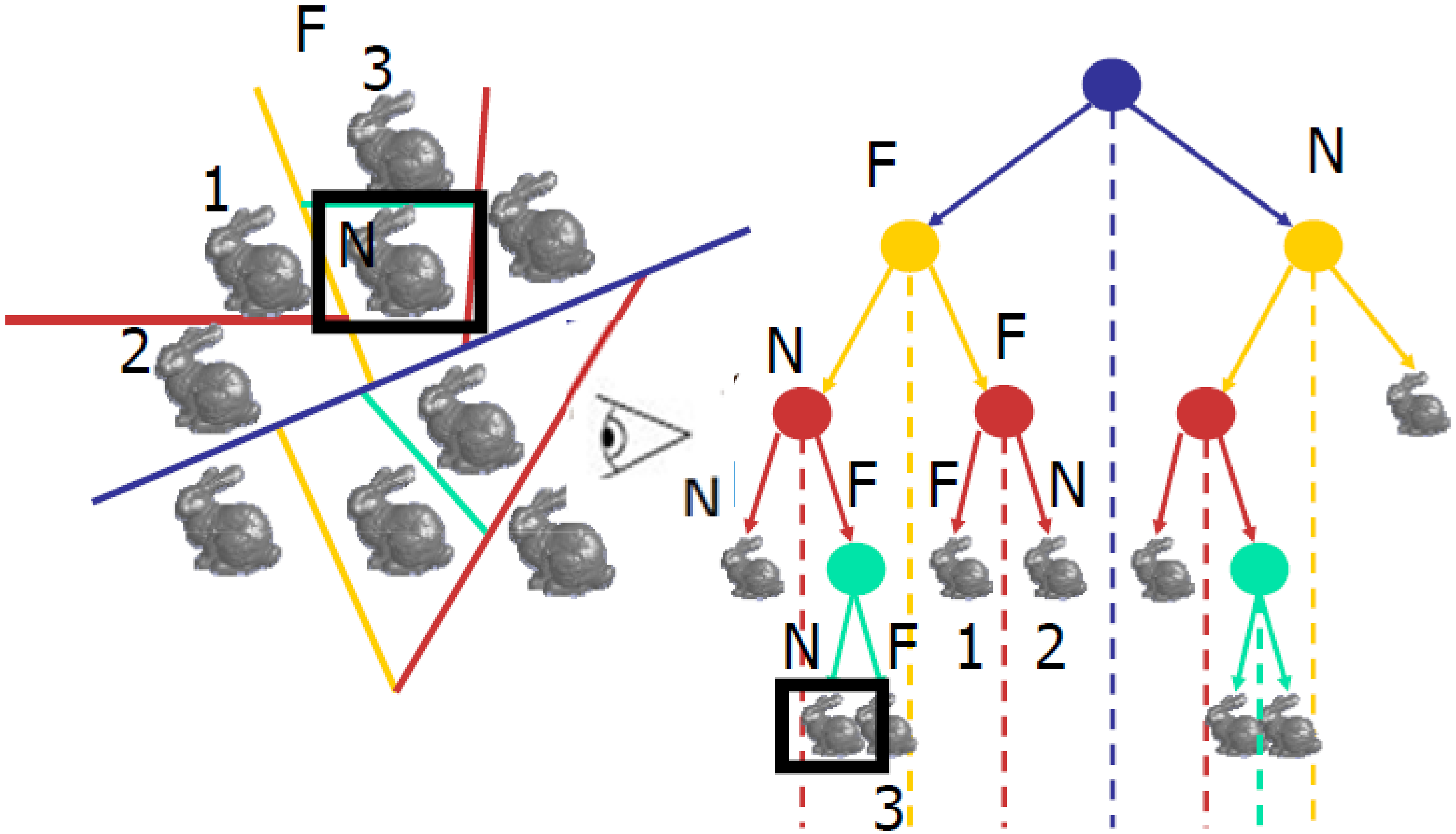
BSP-Trees: Viewpoint A



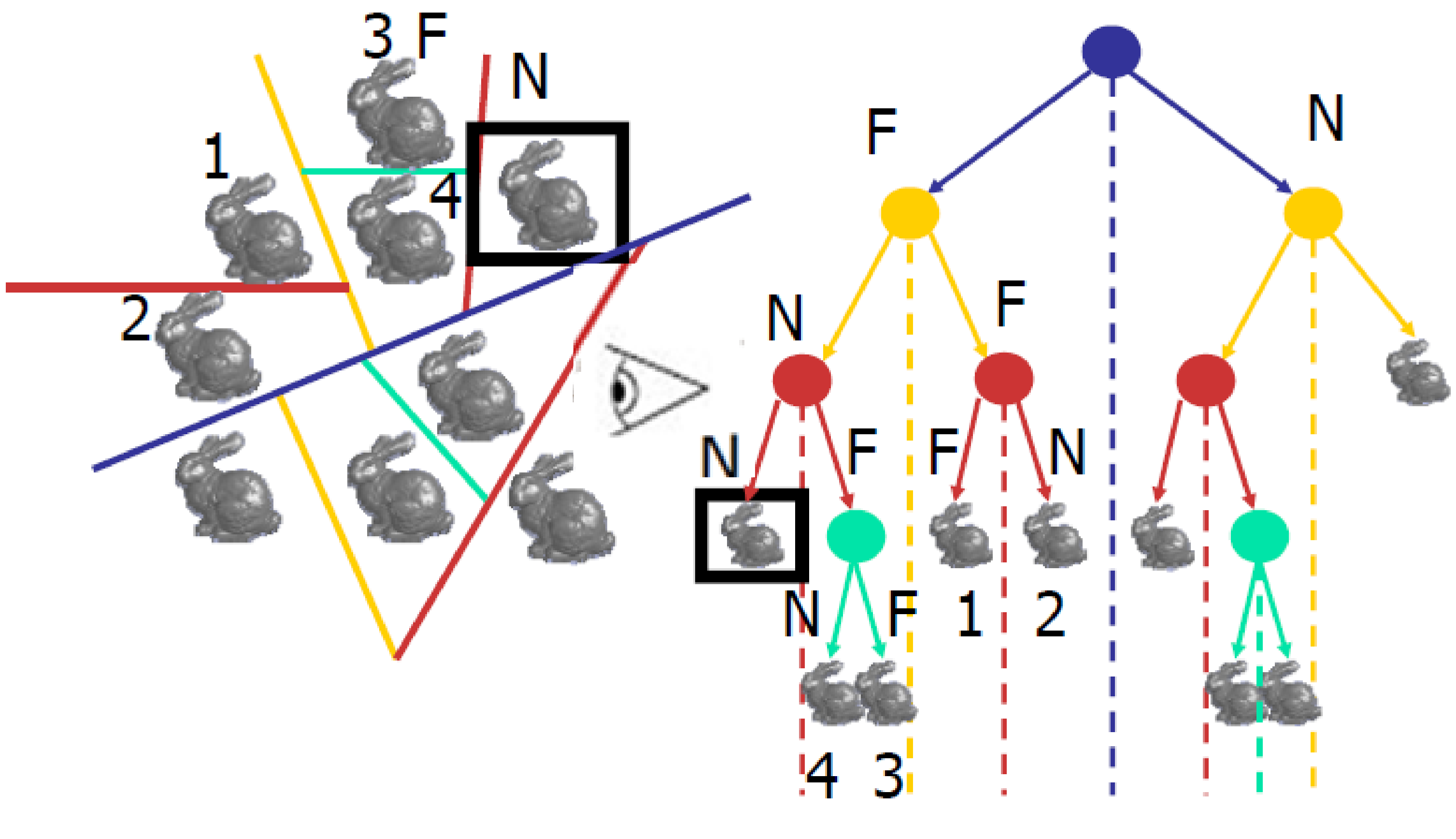
BSP-Trees: Viewpoint A



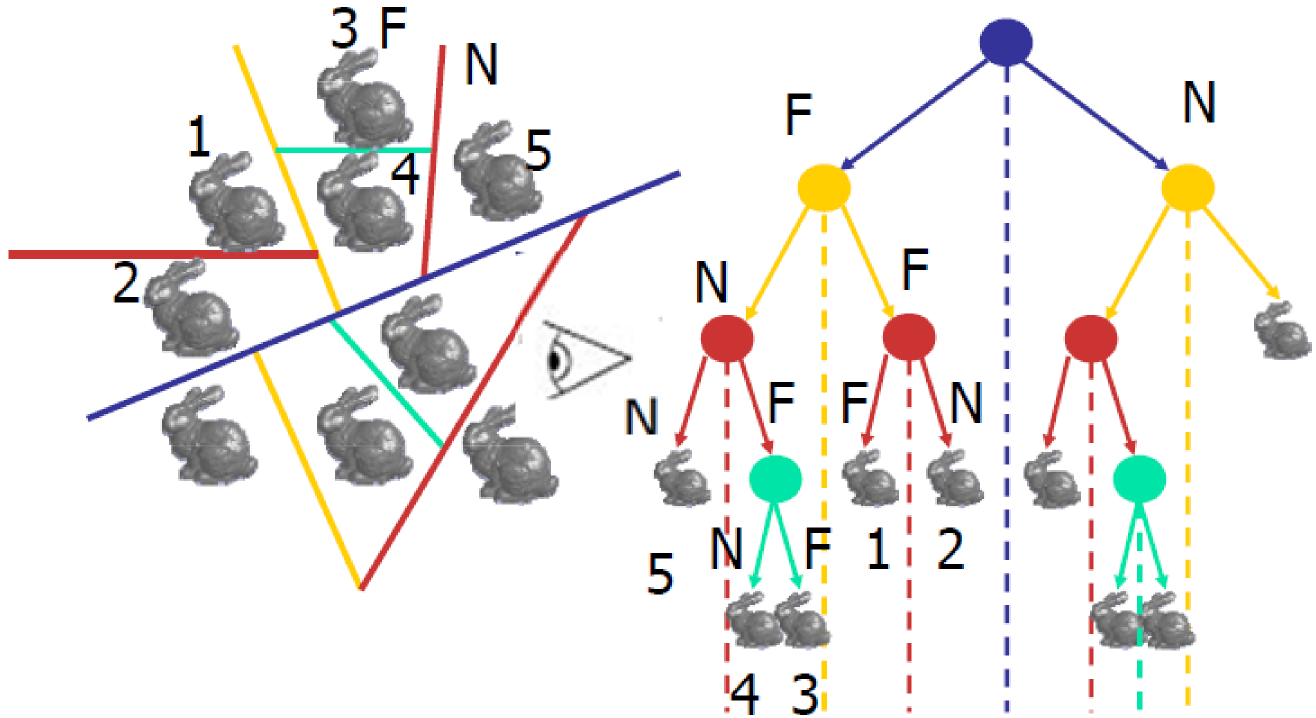
BSP-Trees: Viewpoint A



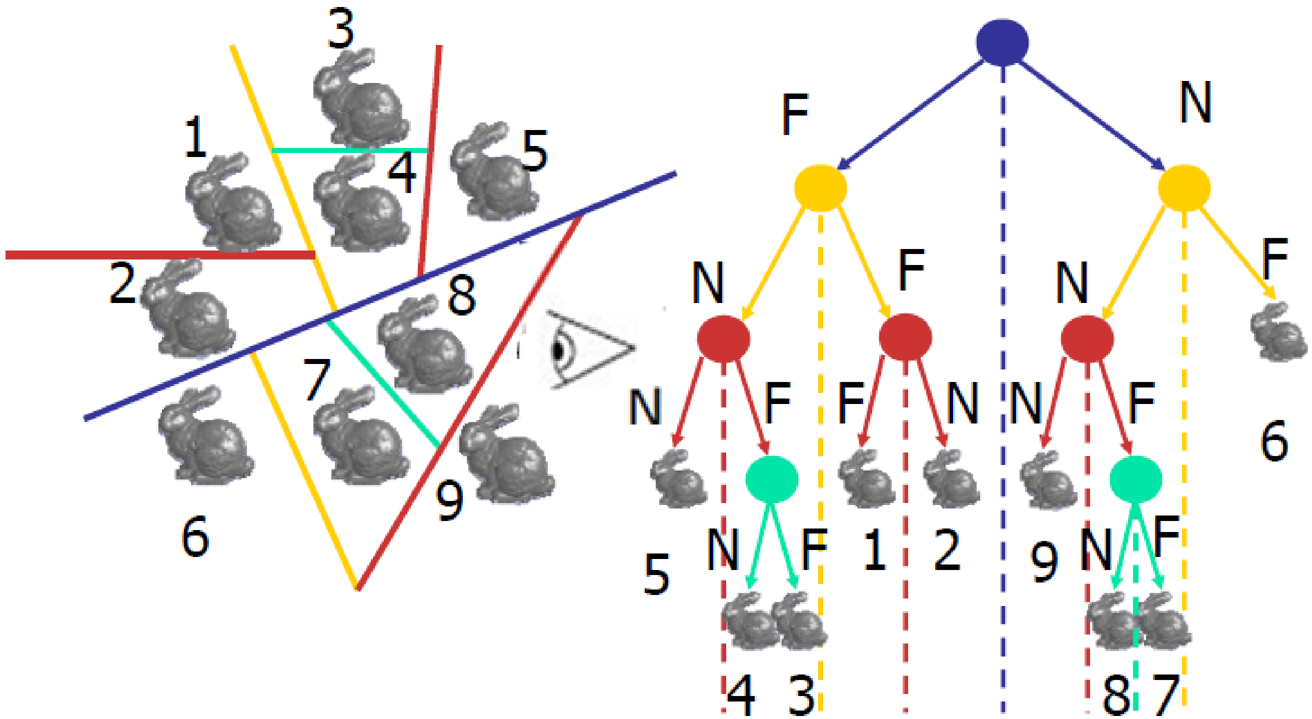
BSP-Trees: Viewpoint A



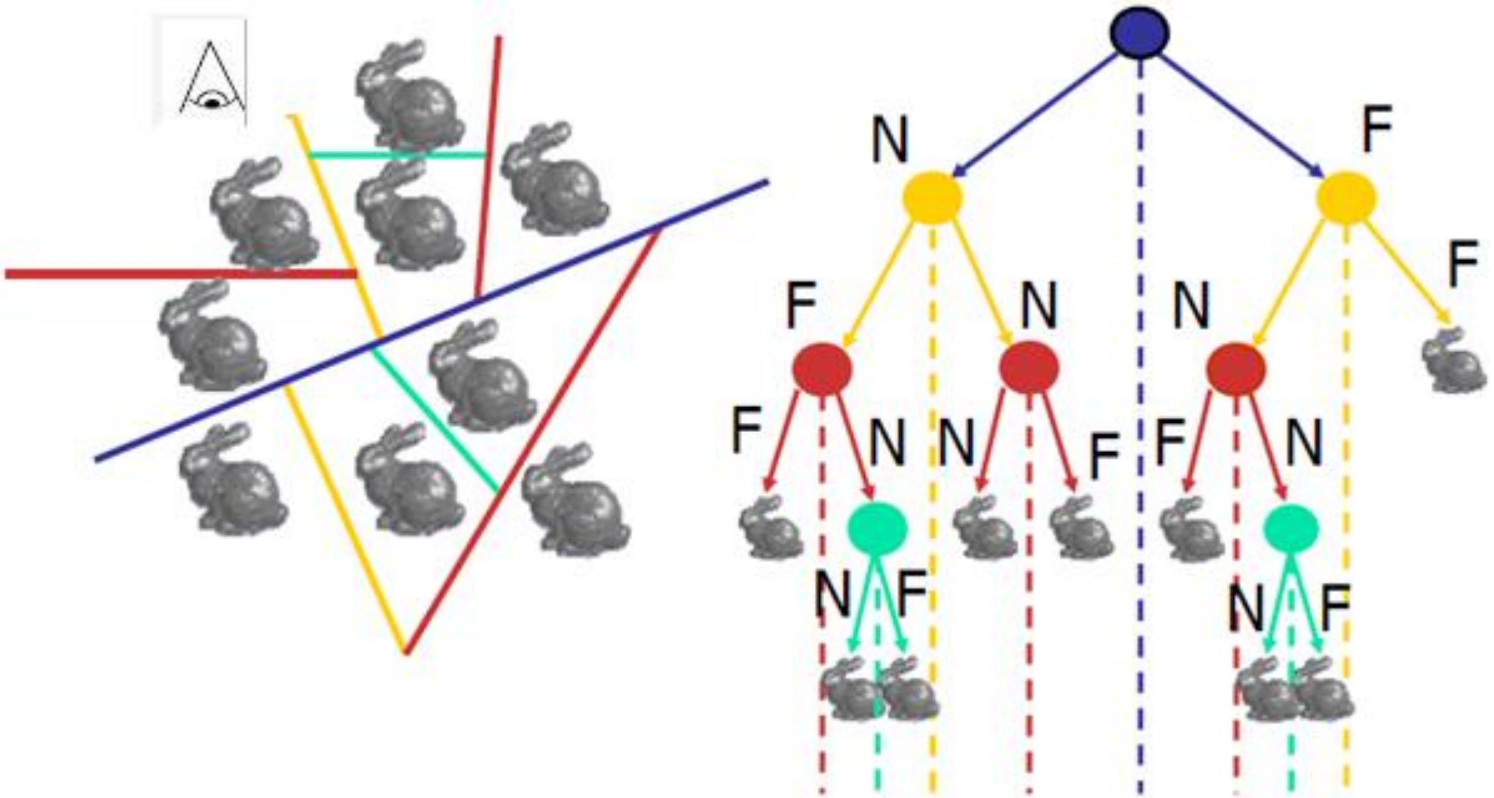
BSP-Trees: Viewpoint A



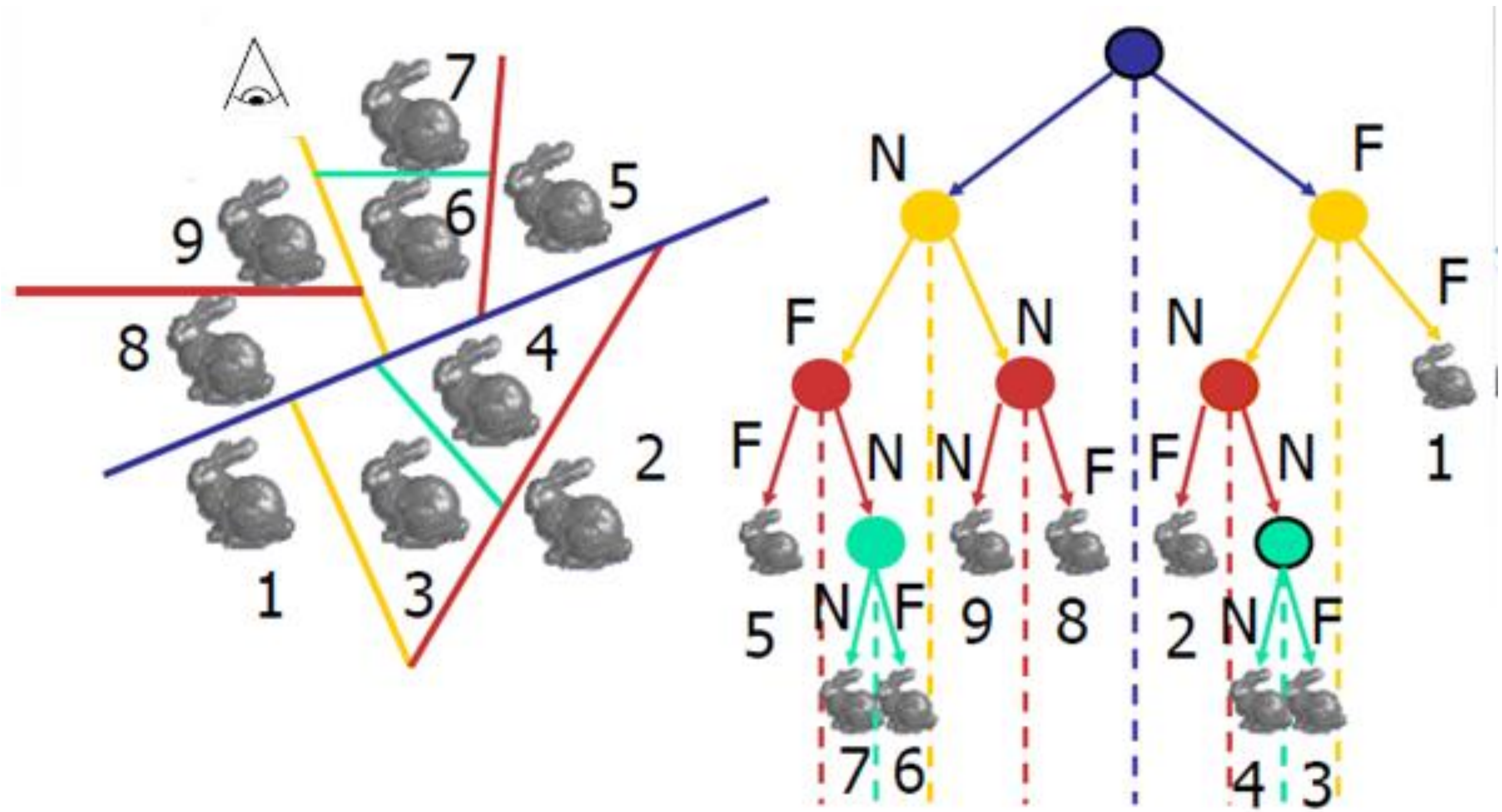
BSP-Trees: Viewpoint A



BSP-Trees: Viewpoint B

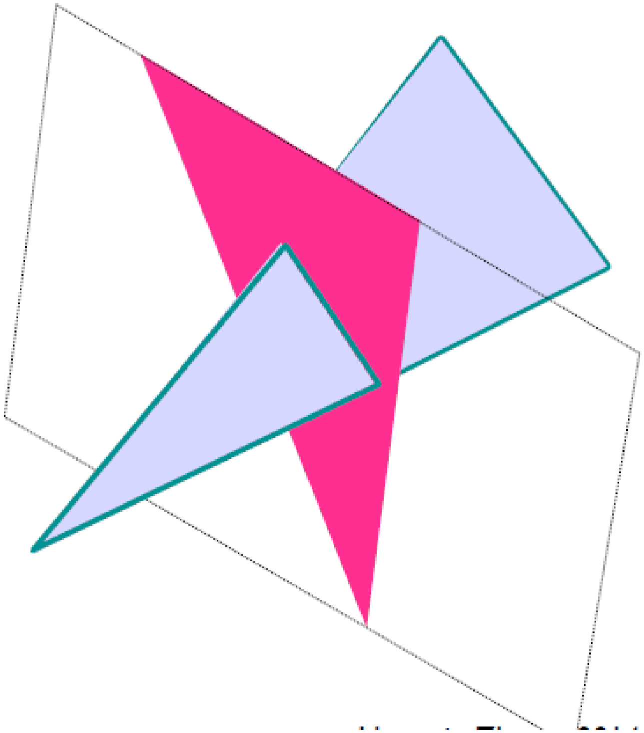


BSP-Trees: Viewpoint B



BSP Tree Construction: Polygons

- The binary tree is constructed using the following principle:
  - For each polygon, we can divide the set of other polygons into two groups
  - One group contains those lying in front of the plane of the given polygon
  - The other group contains those in the back
  - The polygons intersecting the plane of the given polygon are split by that plane



BSP Tree Traversal:Polygons

- Split along the plane defined by any polygon from scene
- Classify all polygons into positive or negative half-space of the plane
  - If a polygon intersects plane, split polygon into two and classify them both
- Recurse down the negative half-space
- Recurse down the positive half-space

Summary: BSP Trees

- Pros:
  - Simple, elegant scheme
  - Correct version of painter's algorithm back-to-front rendering approach
  - Still very popular for video games (but getting less so)
- Cons:
  - Slow(ish) to construct tree: O(n log n) to split, sort
  - Splitting increases polygon count: O(n2) worst-case
  - Computationally intense preprocessing stage restricts algorithm to static scenes

BSP Demo

- **Useful Demo**

  http://www.symbolcraft.com/graphics/bsp/